Dynamics of spin glasses

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1 Introduction

Glass is an extremely familiar material and has been used and produced by mankind for thousands of years. Surprisingly, the complicated cooperative slowing down that takes a glass from its high temperature viscous liquid state to its low temperature non-crystalline solid state is still very poorly understood, even to the extent that its status as a thermodynamic transition has been questioned. It is a continuous transition, like a second order transition, but there is no simple low temperature state being formed which can be recognised as being "ordered". First and second order thermodynamic transitions in regular systems are now extremely well understood, but this is far from being the case for the glass transition. As structural glass transitions are very difficult to think about and model, it is worth while studying carefully the magnetic analogues of glasses, the spin glasses, and in particular their dynamics.

2 Spin glass materials

The first spin glasses which were recognised as such were dilute metallic alloys of a magnetic transition element (Fe, Mn) in a non magnetic host (Cu, Ag, Au). It had long been noticed that these materials had Curie-Weiss like susceptibilities and that they formed some sort of strange "antiferromagnetic" low temperature state. It was not expected that there would be any precise transition temperature, because of the variable local impurity concentrations, and because of the fact that there was no sign of any sharp feature in the temperature variation of the specific heat; in every case only a broad smooth hump had been observed in $C(T)$. However in 1969 it was discovered that low field ac susceptibility measurements showed a sharp cusp as a function of temperature, clear evidence for a well

defined transition temperature T_g (Canella *et al.* 1971). The same year these systems were dubbed "Spin Glasses", following a discussion between Coles and Anderson, (Anderson 1970).

For the next 20 years there was intensive activity in this field. It was realised that spin glasses were of great interest as magnetic examples representative of the much more general class of materials having glassy ordering. Spin glasses could be investigated conveniently both in the laboratory and numerically, because of their magnetic character. Conceptually, thanks to an intense theoretical effort considerable advances were made in the mean field (or infinite interaction length) limit. It was realised that freezing represented a choice by the system of one among many sets of Gibbs states which were not related by symmetry, and an analytically exact solution was obtained (Parisi 1983). Although spin glass alloys *per se* are totally useless for any practical applications, the ideas generated by this work provided extremely important spin-offs: the creation of "neural network" computers, and simulated annealing methods in optimisation problems. However, there is to this day no consensus as to the correct theoretical description of the finite dimensional case. Reviews are given in Binder and Young (1986) and Mydosh (1993).

It was soon found experimentally that the spin glass label could be applied to many systems outside the original dilute alloys, and hundreds of examples of systems in this category have been discovered in all sorts of materials. Two essential conditions for spin glassiness are believed to be

- frustration in the interactions between the spins, and
- a certain degree of disorder.

Large scale numerical simulations have been performed, principally in the Edwards and Anderson (1975) model, which is a theorist's spin glass consisting of Ising spins placed on a regular lattice but having interactions which are random. Perhaps surprisingly, this idealised type of spin glass appears to share all the essential characteristics of the real life laboratory systems. The study of spin glasses and related subjects has become a major preoccupation of the statistical physics community.

One of the phenomena which has been the most extensively investigated experimentally is the slow relaxation of the magnetisation (or magnetic creep) which occurs below the freezing temperature T_g. This is a strongly non-exponential relaxation, which can be approximately parameterised by a log(t) decay of the magnetisation. The time scales for this decay extend from microscopic to essentially infinite times (see Mydosh 1993). This is related to the impossibility for the system to find an equilibrium state in any finite time because of the complexity of the phase space.

It is less widely realised that above T_g also the relaxation in spin glasses is highly anomalous compared with that of standard paramagnets. Experiments (Mössbauer spectra, inelastic neutron scattering, μSR) and simulations show that for a very wide range of temperatures above T_g relaxation is non-exponential and much slower than in the paramagnetic state of a ferromagnet, for instance. The behaviour in spin glasses mirrors that found in structural glasses, where both the creep at temperatures below T_g and the anomalous relaxation above T_g are ubiquitous. The main motivation for the spin glass work is to advance concepturally so as to bring a new light to bear on the behaviour of the structural glasses.

3 Glassy dynamics

It is easier to obtain quantitative information on the dynamic behaviour of spin glasses numerically than experimentally, though even the simulations encounter considerable problems related to the long time scales. Also, because of the disorder, each sample is a different individual and averages have to be made, sometimes over some thousands of samples defined by particular choices of the random interactions, in order to get statistically significant data.

Ogielski (1985) published a remarkable set of large scale simulation data on the dynamics of the Ising spin glass in dimension 3; the spins were placed on a regular cubic lattice and the near neighbour interactions were chosen $+J$or $-J$ at random. Ogielski used samples as big as 64^3 spins and carried out measurements up to 10^7 Monte Carlo Steps per spin. He measured principally the autocorrelation function $q(t) = \langle S_i(0)S_i(t)\rangle$ which is a measure of the degree to which the configuration of the system at time t resembles the initial configuration at time zero. It has a value one at the start of a run, and must always decay to zero at sufficiently long time in the paramagnetic state. In the ordered state it will "never" decay to zero, and this is a fundamental definition of ordering. Ogielski used a standard scaling form to parameterise $q(t)$, which is $q(t) = \lambda t^x f(t/\tau)$. He chose empirically for the scaling function f the stretched exponential

$$f(t/\tau) = \exp\left[-(t/\tau)^\beta\right].$$

The parameters λ, x, β and τ were fitted at each temperature. Ogielski chose the stretched exponential as it is very widely used as a fitting function by the glass community. This function, first introduced a century and a half ago by Kohlrausch (1854) and rediscovered by Walker and Watts (1970), expresses the non-exponential behaviour of a decay through the exponent β.

Ogielski's fits were excellent; he defined a freezing temperature T_g by the divergence temperature of $\tau(T)$. The value of $\beta(T)$ was found to decrease gradually from 1 at high temperature to a value close to 1/3 at T_g. A low value of β corresponds to a wide range of effective relaxation times. It turns out that many other model systems have been found to relax in a similar manner with much the same limiting behaviours. These include different Ising spin glasses (Campbell and Bernardi 1994), the "Facilitated Kinetic Ising model" (Graham *et al.* 1997), a Lennard-Jones model glass (Angelani *et al.* 1998), and a charge density wave system (Erzan *et al.* 1990). In addition laboratory glass former materials such as a colloid glass (Bartsch *et al.* 1992), a range of polymer glasses (Alegría *et al.* 1995), and many others, also show stretched exponential decays with exponents $\beta(T)$ which head towards values close to 1/3 at the temperature where the relaxation time diverges.

It is important to find out if laboratory spin glass systems follow the same pattern.In real spin glasses this has proved difficult because the time scales fall in an awkward range and the strongly non-exponential dynamics mean that a wide time spectrum must be covered. Early neutron spin echo (NSE) work which represented an experimental *tour de force* (Mezei and Murani 1979, Mezei 1983) showed that inded the relaxation in a canonical spin glass is strongly non-exponential just above T_g. Because of intrinsic experimental difficulties it has been difficult to improve on these results. Nevertheless the pattern of NSE and simulation data appear very similar.

4 Muon measurements

μSR can probe slowly relaxing spin systems with a wide relaxation rate spectrum, and the technique has been used very effectively to investigate spin glass relaxation behaviour. Pioneering measurements on **CuMn** first showed qualitatively that there was a rapid increase of muon depolarisation rate in the region of T_g (Murnick *et al.* 1976). The ZF, LF and TF (zero field, longitudinal field, transverse field) depolarisations in **Ag**Mn were analysed by Heffner *et al.* (1982), MacLauchlin *et al.* (1983) and Heffner and MacLauchlin (1984) who estimated the temperature dependence of the spin relaxation time as the critical temperature was approached from above and also discussed the low temperature behaviour. Uemura *et al.* (1985) made an extensive analysis of μSR data below and above T_g in **Au**Fe and **Cu**Mn alloys. They modelised in terms of coexisting static and dynamic random fields and allowed for the distribution of the strength of local dipole fields due to the fact that in dilute alloys there are many inequivalent muon sites. The problem was addressed again by Pinkvos *et al.* (1990).The muon data from these three sets of experiments showed conclusively that certain models for spin glass freezing ("inhomogeneous" models) could be definitively ruled out. The freezing is not a process where some spins became static and then others gradually join them as the temperature is lowered; it is a cooperative effect where all spins develop a static component at the freezing temperature. This is an important contribution to the understanding of spin glass physics.

We can now summarise the behaviour found by these groups in the different ranges of temperature.

First of all, we should note that in these alloys, at temperatures below about 200K the muons can be considered as static and placed at random on interstitial sites in the lattice (Hartmann 1979, Clawson *et al.* 1983, Brown *et al.* 1981). (Only at high temperatures can muon diffusion become important and complicate the analysis). The random and dilute local magnetic spins are interacting with the muons essentially through the dipole-dipole interaction. Luckily, in Ag and Au based alloys host nuclear moments are weak so muon depolarisation by the nuclei can safely be ignored. In Cu there is depolarisation of the muons by the nuclei in the sample; this is a nuisance but it can be eliminated by applying a weak (100G) decoupling field, which has little effect on the intrinsic spin dynamics unless the magnetic site concentration is weak and the temperature is close to T_g.

Let us start from the low temperature range. At temperatures very far below T_g, the local spins are completely frozen into one of the many complex configurations which minimise the magnetic energy. As far as the muons are concerned, the orientations of the spins appear random. This leads to a static Kubo-Toyabe depolarisation pattern of the Lorentzian type (because of the random positions of the spins)(Walker and Walstedt 1980), with the interaction strength distribution breadth proportional to the impurity concentration c at low concentrations (a consequence of the $1/r^3$ form of the dipole interaction). In zero applied field the expression for the average muon polarisation at time t, established by Kubo, is

$$g_z{}^L(t) = 1/3 + \frac{2}{3}(1 - at)\exp[-at].$$

This familiar pattern starts by falling linearly, has a minimum near at=2, and then tends towards the limiting value of 1/3 at times long compared to the interaction strength. In an applied longitudinal field the depolarisation plateau rapidly becomes higher, reaching

about 0.9 when the external field is 10 times the breadth of the internal field (see Uemura *et al.* 1985, Figure 2).

The width of the static field distribution, a, is proportional to the impurity moment. If $\Delta^2_{\max}$ is the hypothetical dipolar width with all the lattice points occupied by spins pointing in random directions ($c = 1$), then the averaged dipole field on a muon in a dilute alloy would be $c\Delta^2_{\max}$. For reasonably dilute alloys $a = \sqrt{(\pi/2)}\, c\Delta_{\max}$ For a **Cu**Mn alloy of concentration 1%, a is $14\mu sec^{-1}$. As one knows the local moment values and because the interactions are dipole, these limiting low temperature breadths can be readily estimated *a priori* for each particular case.

Suppose now that the temperature is increased to some rather higher value, still below T_g. In a regular system (a ferromagnet or antiferromagnet) there would be magnons: cooperative fluctuations of the local moments about their time average positions at characteristic frequencies which are of the order of T and so high compared to the muon frequencies. The local moments would still appear static but with reduced amplitude as compared to zero temperature. In the spin glass on the other hand it turns out that there is a wide spectrum of "magnon" frequencies, with soft magnon frequencies extending down into the muon range and beyond. The existence of these soft magnons was predicted from simulations (Walker and Walstedt 1983); muons probe this low energy excitation spectrum particularly well. Thus the local moments appear to have a reduced amplitude, but there are simultaneously fluctuations slow enough to depolarise the muons. After a fast initial Kubo-Toyabe dip and rise to the 1/3 polarisation level, there is a subsequent decay towards zero at long times which would not exist if the local moments were strictly static. The picture of time average static moments plus fluctuations was convincingly demonstrated to be correct by LF measurements somewhat below T_g. The effect of the applied field on the depolarisation could only be interpreted consistently by this type of model (Uemura *et al.* 1985). The lower the temperature, the slower this decay becomes as the fluctuations freeze out. Data in fields up to $0.5T$ were analysed so as to obtain information on the noise spectrum of the low frequency fluctuations, which was interpreted in terms of a power law in frequency. The muon relaxation rate λ drops off as about $H^{-0.5}$. On general grounds it can be expected that λ is proportional to the noise intensity produced by the fluctuations at the muon Larmor frequency. This implies that below T_g the local spin correlations decay algebraically (Heffner and MacLauchlin 1984). (Of course this analysis concerns the part of the spectrum visible with muons; it should be remembered that standard macroscopic magnetic measurements show that there is still magnetic relaxation extending to essentially infinitely long times below T_g). The temperature evolution of the static spin amplitude follows a Langevin like curve, similar to that seen in Mössbauer measurements on spin glasses (Uemura *et al.* 1985).

As well as standard spin glasses, we can also consider "reentrant" systems below the Curie temperature. In an alloy series (for instance **Au**Fe, but there are many others) which is spin glass at one end of the concentration range and ferromagnetic at the other, there is a critical concentration above which ferromagnetism sets in. However within the ferromagnetic region, ordering is not simple. As the temperature is decreased for fixed concentration, there is first a standard Curie temperature T_c with onset of ferromagnetic ordering. Then there is a "canting temperature" T_k where the spin components perpendicular to the local domain magnetisation direction freeze, without the loss of ferromagnetic order for the components parallel to the domain magnetisation. Finally, below this tem-

perature low energy fluctuations appear (Hennion *et al.* 1984, 1986). The canting has been carefully demonstrated by neutron diffraction and other methods. Muon relaxation measurements show conventional slowing down of fluctuations at T_c with behaviour becoming quasi-static as the temperature drops. However around a temperature T_F *below* T_k there is a striking increase in the depolarisation rate (Barsov *et al.* 1994 Mirebeau *et al.* 1997), which is manifestly related to very slow fluctuations. It is curious that these strong slow excitations appear below the canting temperature, rather than accompanying the ordering as is the case at the Curie temperature. Remarkably, the complex sequence of transitions was predicted theoretically at a very early stage (Gabay and Toulouse 1981).

If the temperature is raised still further, as soon as we are above T_g there are no static components of the local moments, and the depolarisation is purely dynamic. Because of critical slowing down, spin relaxation is very slow right at T_g and becomes much faster as the temperature increases. As a first approximation, it is convenient to assume that all the different local spins are relaxing at the same rate with a characteristic time τ. The depolarisation effect on the muons however varies from muon site to muon site, because some muons are closer to a local spin (and so are submitted to a strong fluctuating dipole interaction) and some are in a region where there are no local spins nearby. In the limit of high dilution of the local spins, the integral of the different depolarisation rates for the different muons leads to a form for the muon depolarisation which is a "root exponential" in the fast modulation limit $a\tau < 0.2$ (MacHenry 1972, Uemura 1981, Uemura *et al.* 1985)

$$P(t) \propto \exp\left[-(\lambda t)^{1/2}\right]$$

with $\lambda = 4a^2\tau$. Curves calculated for different values of $a\tau$ are given in Uemura *et al.* (1985), Figure 4. The depolarisation behaviour observed experimentally in spin glasses somewhat above T_g is indeed close to this form but we will see below that other mechanisms must be taken into account. (The geometrical effect appears to account very well for data taken in a superparamagnetic system of small dilute Co clusters in Cu; Bewley and Cywinski 1998.) The effective local spin relaxation time $\tau(T)$ changes dramatically close to T_g, from 10^{-8} very near T_g to about 10−12 sec by $2T_g$. If it is assumed that $\tau(T)$ has standard critical behaviour, then the observed divergence goes as $(T-T_g)^n$. The value found for the exponent n is about 2.8 (Uemura *et al.* 1985, Pinkvos *et al.* 1990). This experimental value is actually considerably smaller than the exponent found in the Ising spin simulations where the characteristic time diverges as about $(T-T_g)^7$ (Ogielski 1985). It is still not clear if this difference is intrinsic. At temperatures very far above T_g, interactions between spins will become unimportant compared with interactions between each spin and the bath, and the relaxation should then be due only to some intrinsic relaxation mechanism. The rate should become concentration independent with a dependence on temperature characteristic of this intrinsic mechanism. For instance, the interaction with the conduction electrons in a metal should lead to a high temperature Korringa relaxation in $1/T$. In addition, at high temperatures there may well be an onset of muon diffusion, leading either to motional narrowing or to muon trapping by the magnetic impurities (Campbell *et al.* 1994). This high temperature region has not been explored in detail.

In the fast fluctuation region, $\omega_L\tau \ll 1$ where ω_L is the Larmor frequency of the muon in the longitudinal field, longitudinal fields have no effect on the muon depolarisation (Uemura *et al.* 1985). In a strong transverse field however the depolarisation rate $\lambda(H)$ can increase considerably (Heffner *et al.* 1982). This is a static inhomogeneous broadening

effect. The spin glass has a finite susceptibility so in an applied field the local moments are partially polarised. This polarisation varies from point to point depending on the local environment, so in consequence the static dipole field at the different muon sites also varies. The Larmor precession rates of the muons are distributed, so λ increases with field. This spin-spin interaction effect is still observed far above T_g (Heffner *et al.* 1982).

5 Non-exponential depolarisation above T_g

Can this analysis be pursued further ? In the discussion sketched out above, it was explicitly assumed that there is a unique local moment relaxation rate at a given temperature, the same for all local spins, and that this relaxation is exponential. Both assumptions can be questioned. The relaxation rate may well vary strongly from spin to spin, because in an alloy the local environments of individual spins are not the same (Pinkvos *et al.* 1990). The instantaneous relaxation rate of any one given spin may well depend on the configuration of its neighbours at that instant; in other words the relaxation of a given spin may be time dependent and so not exponential. Again, if we appeal to simulations on a model system, where the orientations of individual spins can be recorded, both effects are observed (Komori *et al.* 1995). The details of the relaxation behaviour in real systems, including these two effects, are of great interest but the difficulty is to extract them from the data integrated over all muons, which is the information provided by the experiment.

Experiments on moderately concentrated spin glasses (5 to 10% magnetic sites) show depolarisation functions above T_g which can be fitted very satisfactorily by stretched exponentials,

$$P(t) \propto \exp\left[-(\lambda t)^{\beta}\right]$$

where λ is again a depolarisation rate and β is an exponent, both being temperature dependent. While λ increases when the temperature is lowered towards T_g as expected, β drops from a value near 1 at high temperatures to a limiting value near 1/3 as T_g is approached (Campbell *et al.* 1994). This behaviour seems to be very general and a number of other spin glasses have been found to follow the same pattern (Cywinski and Rainford 1994, Dunsiger *et al.* 1996, Telling *et al.* 1998, Stewart *et al.* 1998). The purely geometrical mechanism leading to the "root exponential" behaviour discussed above cannot be an adequate explanation. First of all, as the alloys are relatively concentrated almost all muon sites have at least one local moment close by. There is no tail of sites with very weak links to any local moment and hence with very slow depolarisation rate. Secondly, this mechanism should lead to a temperature independent stretching exponent β, which is not what is seen. One is forced to conclude that the behaviour observed is a signature of the effects invoked in the last paragraph - a wide spectrum of local spin relaxation rates, combined with non-exponential relaxation at individual sites. When β tends to about 1 at high temperatures, relaxation has become "normal": to a good approximation, a regime with a unique, exponential relaxation for all spins has set in. When β takes on much lower values, it is the sign of a widening of the relaxation spectrum specific to spin glasses. What is remarkable is the degree of universality of this behaviour. A limiting value of β of approximately 1/3 when T_g is approached has been observed in an impressive number of systems. The crossover to exponential depolarisation always occurs well above T_g, generally at temperature 5 to $10T_g$. Behaviour of this type has thus been observed in a

range of canonical metallic spin glasses (Campbell *et al.* 1994), in unorthodox metallic spin glasses (Cywinski and Rainford 1994, Telling *et al.* 1998, Stewart *et al.* 1998) in insulating spin glasses (Campbell *et al.* , unpublished) , and even in the particular case of a pyrochlore spin glass (Dunsiger *et al.* 1996). The implication is that there is a universal form of dynamics, with its associated temperature dependent time spectrum, which is a necessary consequence of spin glass ordering.

It would be most instructive if it were possible to invert the information, and to go from the observed form of the muon polarisation decay to the local spin autocorrelation function. This can be done if we make the assumption (which may not be justified) that the dynamics of the spin glass can be treated in terms of a distribution of individual local effectively exponential relaxation processes. The depolarisation rate of a muon at a given site is then proportional to the relaxation time of the local spins in its immediate environment. We can write a Laplace transform of a muon depolarisation function to obtain the distribution of effective local spin relaxation times, and then integrate over these to find the local spin autocorrelation function. Even if we consider that the stretched exponential depolarisation is only a convenient fitting approximation with no fundamental significance, it is clear that a low value of β in the depolarisation decay will transform to a wide distribution of relaxation times and hence to a strongly non-exponential autocorrelation function relaxation. Explicit calculations are were made by Campbell *et al.* (1994). If we assume that each muon i has an exponential depolarisation rate λ_i, the form of the overall depolarisation pattern will reflect the distribution of the λ_i through a Laplace transformation. The particular distribution

$$G(\lambda_i) = \frac{\lambda^2}{2(\pi\lambda_i^3)^{1/2}} \exp(-\lambda^4/4\lambda_i)$$

will give a stretched exponential depolarisation with $\beta = 1/2$, i.e.

$$g(t) = \exp\left[-(\lambda t)^{1/2}\right]$$

Now suppose we consider that the depolarisation of each muon i reflects the relaxation of a nearby spin, with a relaxation time τ_i proportional to λ_i. The global autocorrelation function $q(t)$ will then be given by the integral

$$\int_0^\infty G(\lambda_i) \exp\left[-t/\alpha\lambda_i\right] d\lambda_i$$

where $\tau_i = \alpha\lambda_i$. The transform leads to

$$q(t) = (1 + 4t/\alpha\lambda^4)^{-1/2}.$$

This is a strongly non-exponential relaxation, even though the transform of a stretched exponential does not give back another stretched exponential.

This treatment of the muon depolarisation data probably gives the best available information obtainable up to now on the relaxation of spin glasses in the paramagnetic state. It is of interest to attempt to go further by exploiting the possibilities of measurements in applied fields. If we ignore the direct effect of the field on the spin dynamics, then the form of the depolarisation in a field $\mathbf{B}$ is directly related to the power spectrum of the dipole field fluctuations at the muon as noted above (Heffner and MacLauchlin

1984). Suppose that we assume that different moments all have the same functional form of relaxation $q(t)$ (for instance an exponential, a stretched exponential, a power law, *etc.*) but that for each local spin i there is a prefactor c_i, with a distribution of values of c_i (Keren *et al.* 1996). Then it can be shown that the in-field muon polarisation, despite a distribution of dipole interaction strengths and relaxation times at the sites i, follows a scaling rule asymptotically. The rule is $g(H,t) = G(t/h^{\gamma})$ where $g(H,t)$ is the polarisation under longitudinal field H and after time t. If $q(t)$ is a power law, i.e. $q(t) \propto t^{-\alpha}$, then $\gamma = 1-\alpha$ which is exactly the relation derived in Heffner and MacLauchlin(1984). If $q(t)$ is a stretched exponential, $q(t) \propto \exp\left[-(\lambda t)^{\beta}\right]$, then $\gamma = 1-\beta$. Experiments show that good asymptotic scaling can be obtained at temperatures near and above T_g (Keren *et al.* 1996). In the future it should be possible to develop this approach so as to exploit all the information implicitly contained in the field dependence of the depolarisation curves so as to recover detailed information on the time dependence of the local spin relaxations.

Other experiments can give independent information on the ordering and the dynamics. in spin glasses. In Fe containing samples, Mössbauer measurements provide a very direct means for observing the effective static local moments as a function of temperature below T_g. Agreement with muon measurements is good (Uemura *et al.* 1985). Above T_g, Mössbauer linewidth can give information on τ (Meyer *et al.* 1986). This is a measurement directly at the magnetic sites, but it turns out that the linewidth becomes negligible compared to the static linewidth as soon as τ is shorter than about $10^{-}10$ sec, so the muon technique has a much wider dynamic range as well as being adaptable to any spin glass. The "ideal" dynamic experiment is Neutron Spin Echo (NSE), which is a direct measurement of the same time dependent spin autocorrelation function as that measured in the simulations(Mezei and Murani 1979). Unfortunately, because of signal intensity considerations, NSE experiments on spin glasses are hard and lengthy.

It should also be mentioned that other systems which are not *bona fide* spin glasses show similar muon depolarisation patterns, characteristic of a spectrum of depolarisation rates which is narrow at high temperatures and becomes broad as the temperature is lowered. The compound DyAg in its amorphous form, a ferromagnet below $18K$, behaves much like a spin glass (Kalvius *et al.* 1986) whereas the crystalline DyAg compound shows exponential depolarisation down to its antiferromagnetic Néel point. The $TbNi_2B_2C$ compound (Hillier *et al.* 1998), which orders as a planar incommensurate antiferromagnet at 15.5K, shows analagous behaviour. In the case of the amorphous compound, a wide distribution of crystal fields might be invoked to explain the broad spectrum of relaxation rates. In the Tb compound the anomalous dynamics might be associated with the complicated ground state.

6 Glassy dynamics and phase space in spin glasses

What is the physics behind the apparently universal dynamic behaviour in glasses and spin glasses? A model has been proposed in terms of phase space morphology (Campbell *et al.* 1987).

The global phase space of a system consists of all its possible configurations or microscopic instantaneous states. For example, the phase space for N atoms of an ideal gas is a space of dimension $6N$ because there are three space coordinates and three velocity

components for each atom. The phase space of a set of Ising spins is simpler. For a system consisting of three Ising spins all the eight possible microstates can be mapped onto the corners of a cube. For an N-spin Ising system the configurations can be mapped exactly onto the 2^N corners of an N-dimensional hypercube; this is the global phase space. Turning over one spin is equivalent to a step from one microstate (i.e. one corner) to a near neighbour microstate. This remains true whatever N.

Relaxation consists in flipping spins; the point representing the instantaneous state of the system will wander from one microstate to another microstate on the hypercube. When there are interactions between the spins, different microstates will have different energies. Thermodynamics tell us that at low temperature only the low energy microstates will be thermodynamically accessible, so the system will wander within this restricted set of microstates. If the interactions between the spins are purely ferromagnetic, the lowest energy microstates will be those with all spins up or with all spins down; at low temperature the set of accessible microstates will be split up into two disconnected clusters: one with mostly spins up (near the bottom left-hand corner of the cube) and one with mostly spins down (near the top-right hand corner). When the system is heated, more and more higher energy states become accessible until the two clusters coalesce at the Curie temperature.

For a spin glass things are more complicated with many possible low-energy ground states. Suppose that the thermodynamically accessible configurations at a given temperature are distributed completely at random over the hypercube. This is too extreme a hypothesis for a real system but it allows us to make progress. The problem is now mapped onto a purely geometrical representation.

Imagine a high-dimensional hypercube with probability p that any individual microstate is accessible. All the other corners are inaccessible. (p plays the role of the temperature; the lower p is, the fewer configurations that are accessible). For this hypercube geometry it has been proved mathematically that below a critical p value, p_c, there occurs the closed-space analogue of a percolation transition Thus for $p > p_c$ there is a giant cluster of linked corners while for $p < p_c$ there exist only isolated small clusters.

One such property concerns random walks. If an ant performs a random walk on the percolation cluster, the average of the square of the distance L from its starting point increases with time according to $\langle L^2 \rangle \propto t^{\beta}$, rather than $\propto t$ as in a conventional random walk. This is because on the percolation cluster the ant wastes time going along dead ends. If the dimension is greater than 6, β is equal to precisely 1/3.

To make contact with relaxation we must imagine an ant representing the instantaneous configuration of the spins wandering in phase space. We set the ant off on a random walk on the giant cluster for a hypercube with sites occupied with probability p. As time increases, the ant moves around the sites and loses memory of its starting point, the initial configuration. It turns out that on the hypercube, the equivalent of a random walk with exponent β in an Euclidean space is a memory function in the form of a stretched exponential with Kohlrausch exponent β. At high p relaxation is rapid and exponential ($\beta = 1$); as p tends towards p_c the random-walk memory-function time scale tends to diverge, and the Kohlrausch exponent beta tends towards 1/3.

We conclude that the observed stretched-exponential relaxation behaviour in an Ising spin glass signifies that its phase space resembles the randomly occupied hypercube. We

make the heuristic ansatz that the spin glass transition can be thought of as a percolation transition in phase space.

The phase space of a polymer glass former is much more difficult to imagine, but a phase space there must be. We can argue from the similarity between the relaxation behaviours of the idealised spin glass and of the polymer glass that the overall morphology of the glass phase space as the critical temperature is approached is of the same type.

This leads us to the following image. Below the temperature of the transition the equilibrium phase space is an archipelago of isolated clusters of configurations in phase space; the system must stay marooned within one particular cluster (different after each quench) and so behaves as frozen . In contrast, well above the transition the system would completely lose memory in a short time because it can sample enough of phase space, and so it can be considered to be a liquid.

In this approach, as a material is cooled towards a glassy freezing temperature, phase space always becomes sparse and fractal-like; the system takes longer and longer to explore enough of phase space to decide if it is still above the glass transition or not. The slow stretched exponential relaxation is a necessary consequence of this complex phase space.

7 Conclusion

The freezing transition and the low temperature behaviour of glasses and spin glasses continue to pose fundamental and difficult physical problems. Spin glasses are easier to conceptualise, to simulate numerically, and to study experimentally. The key to understanding these systems certainly lies in the dynamics. Muons turn out to be probes which are very well adapted to the study of spin glasses, because of the time window of magnetic fluctuations that they are sensitive to. By the application of a longitudinal field it is possible to scan the spectrum of the fluctuations. The wide range of experiments already performed have provided important information on the details of the dynamics above the transition and the anomalous behaviour below the freezing point. Future work with more sophisticated analyses and detailed comparisons between different systems should provide conclusive insight in this domain.

8 References

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