Universal correlations and condensation mechanisms in unconventional superconductors

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1 Introduction

Over the last decade transverse field muon spin rotation has proved to be an extremely powerful and sensitive microscopic probe of the internal field distribution within the mixed state of bulk Type II superconductors (Lee, this volume). It offers what is perhaps the most direct and accurate method of measuring the superconducting penetration depth and coherence length whilst also circumventing many of the intrinsic problems associated with, for example, bulk magnetisation measurements. Consequently μSR has been widely used in systematic studies of vortex lattice structures, particularly in the high temperature cuprate superconductors and other exotic systems. In the early stages of such μSR studies a rather surprising universal scaling relationship was soon recognised (Uemura et al 1989). It was observed that, for the high temperature cuprate superconductors, an initial increase in carrier doping leads to a linear increase of T_c with muon spin depolarisation rate, σ. Remarkably, as shown in Figure 1, Chevrel phase compounds, bismuthates, fullerenes, organic superconductors and heavy fermion systems were soon found to exhibit a very similar scaling behaviour (Uemera et al 1991, Uemura 1991,1997). All of these systems have been loosely classified as "exotic" or "unconventional" superconductors on the basis of their short coherence lengths, long penetration depths, high upper critical fields and highly correlated electronic structures. It is interesting to note that the transition temperatures of the so-called "conventional" BCS superconductors, such as Nb, do not follow the same scaling relationship.

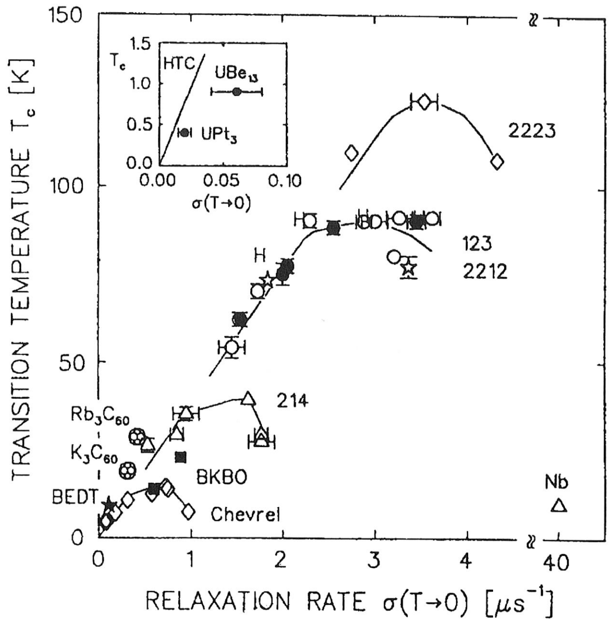

Figure 1. *A plot of the superconducting transition temperature, T_c, against the muon spin depolarization rate, σ (extrapolated to $T = 0$) from Uemura (1997)*

2 Implications of the universal scaling

The implications of this scaling relationship become clearer with the appreciation that the depolarization rate, $\sigma(T = 0)$, is related, via the zero temperature limit of the penetration depth, $\lambda(0)$, and the general London formula, to the two fundamental parameters of the superconducting ground state, namely the effective electron mass, m^*, and the superconducting carrier density, n_s. We find that

$$\sigma(0) \propto \frac{1}{\lambda(0)^2} = \frac{4\pi\, r_e m_e n_s}{m^*} \frac{1}{1 + \xi/l_e} \tag{1}$$

where $r_e(=2.82\times10^{-15}\mathrm{m})$ is the classical radius of the electron, l_e is the electron mean free path, and ξ is the superconducting coherence length. ξ/l_e defines the dirty limit correction. Correspondingly we can see that, within the clean limit, ie for $\xi/l_e \ll 1$,

$$\sigma(0) \propto \frac{n_s}{m^*} \tag{2}$$

The linear scaling relationship between T_c and $\sigma(0)$ observed by Uemura and co-workers (1989, 1991,1997) and illustrated in Figure 1 thus implies a direct correlation between T_c and n_s/m^*. Such a linear correlation is not consistent with the conventional weak coupling limit of BCS theory in which the electron pairing mechanism is phononic in origin. Here the Debye frequency, ω_D, defines the energy scale of the pairing such that $T_c \propto \omega_D$. Usually the electronic density of states is structureless on the scale of $\hbar\omega_D$ and so T_c is not related in any obvious way to n_s.

A clue to the origin of the scaling relation emerges if we recognize the quasi-two dimensional character of both the high T_c cuprates and the organic superconductors. For a non-interacting 2d electron gas the effective Fermi energy is given by

$$E_F = k_B T_F = (\hbar^2 \pi) n_{s2d} / m^* \tag{3}$$

where T_F is the effective Fermi temperature and n_{s2d} is the carrier concentration within the superconducting planes (which may be estimated from the volume carrier density using, dn_s, in which d is the interplanar spacing). The linear correlation between T_c and n_s/m^* thus implies (Uemura et al 1991, Uemura 1991, 1997) that, for these systems at least, $T_c \propto E_F$ and hence $T_c \propto T_F$. Such a relationship is expected only if the energy scale of the electron pairing in the superconducting state is comparable to, or exceeds, E_F (Emery and Reiter, 1988). It is instructive to reformulate Figure 1 as a plot of superconducting transition temperature against Fermi temperature. For the quasi-2d systems T_F may be estimated directly from the muon depolarisation rate $\sigma(0)$ using Equation (3). For a 3d system, however, the Fermi temperature is given by

$$k_B T_F = (\hbar^2/2)(3\pi^2)^{2/3} n_s^{2/3} / m^* \tag{4}$$

and to extract T_F the measured muon depolarisation rate must be coupled with, for example, the Sommerfeld constant, γ, for which

$$\gamma = \left(\frac{\pi}{3}\right)^{2/3} \frac{k_B^2 m^* n_e^{1/3}}{\hbar^2} \tag{5}$$

where n_e is the carrier density. Assuming that, at $T = 0$, n_s is equivalent to n_e above T_c we may combine Equations (2), (4) and (5) to obtain

$$k_B T_F \propto \sigma(0)^{3/4} \gamma^{-1/4} \tag{6}$$

where it can be seen that the estimated T_F is relatively weakly dependent upon γ. Using this parameterization a close correlation between T_c and T_F for those systems generally classified as exotic or unconventional superconductors can be readily confirmed. The cuprate, heavy fermion, organic, fullerene and Chevrel phase superconductors all follow a similar linear trend with $1/100 < T_c/T_F < 1/10$, in contrast to the conventional BCS superconductors (Nb, Sn, Al etc) for which $T_c/T_F < 1/1000$ as illustrated in the plot of $\log(T_c)$ against $\log(T_F)$, shown in a stylized form in Figure 2. On the basis of this plot, now widely known as the Uemura plot, it is tempting to place all of the "exotic" superconductors in a single "class" which appears to be quite distinct from the class of conventional BCS superconductors.

3 Evidence for Bose-Einstein condensation

The implications of the observed scaling behaviour are therefore really quite profound. Indeed this scaling is perhaps an indication that the strongly coupled "exotic" superconductors may, in a thermodynamic sense, be close to Bose-Einstein condensation . The condensation temperature of an ideal boson gas is defined only by n_s and m^*, and is independent of the scale of the pairing interaction, providing that $\hbar\omega_B \gg kT_B$ (Mincus et

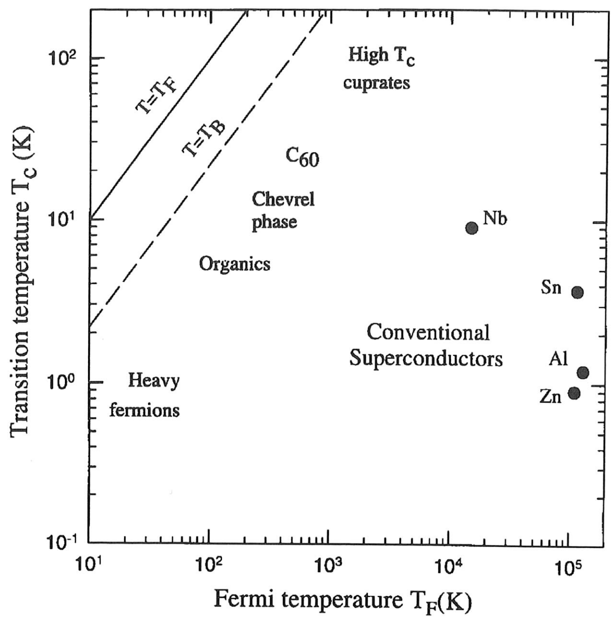

Figure 2. *A schematic representation of the "universal" Uemura plot (after Uemura 1991, 1997) of superconducting transition temperature (T_c) against Fermi temperature (T_F). The shaded region indicates the linear regime over which T_c scales with T_F for the exotic superconductors.*

al, 1990). The BE condensation temperature, T_B, represented graphically by the dashed line in Figure 2, has been estimated using the expression for an ideal 3d boson gas, ie

$$k_B T_B = (1.04\hbar^2) n_B^{2/3} \big/ m_B \tag{7}$$

where it assumed that the boson density is $n_B = n_s/2$ and the boson mass is $m_B = 2m^*$. (Although there can be no Bose-Einstein condensation in a perfect 2d system the resulting value of T_B nevertheless provides an estimate of the maximum condensation temperature for the quasi-2d systems discussed here). All the exotic superconductors are thus found to have values of T_c/T_B in the range 1/3 to 1/30, thereby emphasizing the proximity of these systems to BE-like condensation. It therefore appears that the Uemura plot of Figure 2 provides a practical method of classifying superconductors in terms of their proximity, on the one hand, to BE condensation (in real space, with non-retarded strong coupling) and, on the other hand, to BCS condensation (in momentum space with retarded weak coupling).

An interpolation between the BE and BCS limits can be effected with a relatively simple model (Uemura, 1997) in which an attractive interaction between charge carriers is mediated by an exchange boson with an energy scale defined by $\hbar\omega_B$. In the limit of dilute carriers, pairs will be formed and the resulting bosons will undergo condensation at a very low temperature, T_c, which will follow the behaviour of T_B in the low density region. In the high carrier density limit, where many carriers overlap, BCS-like behaviour is expected where pair formation and condensation occur at the same temperature, T_c.

Again, in this region T_c is expected to be quite low. Between these two limits we might expect a phase diagram similar to that shown schematically in Figure 3, in which a crossover from BE-like to BCS-like behaviour occurs at a carrier concentration where the effective Fermi energy, $E_F = k_B T_F$, is comparable to the energy scale, $\hbar\omega_B$, of the exchange boson. The crossover region thereby separates the non-retarded interaction at low carrier densities from the retarded interaction at high densities. Optimal transition superconducting transition temperatures are expected for $k_B T_F \approx \hbar\omega_B$.

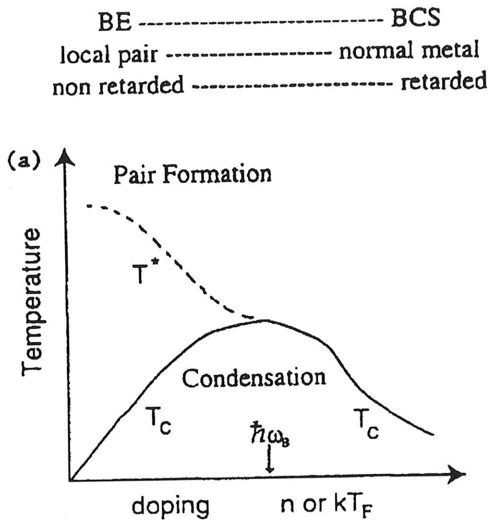

Figure 3. *A schematic representation of the superconducting phase diagram interpolated between the low carrier density BE condensation limit and the high carrier density BCS limit Also illustrated is the suggested carrier density dependence of the pseudogap temperature T^* (from Uemura, 1997).*

4 Underlying condensation mechanisms

Based upon the above observations it is tempting to suggest that the condensation mechanisms in the extraordinarily diverse group of exotic superconductors may share a common origin. Indeed, there is already a growing consensus that the origins for the strong coupling observed in the exotic high T_c and heavy fermion superconductors may be found in dynamical spin fluctuations close to magnetic instabilities (Mathur et al 1998). Recent detailed calculations within Moriya's extremely successful self consistent renormalisation (SCR) theory of spin fluctuations (Nakamura et al, 1996) have suggested that antiferromagnetic spin fluctuation mechanisms may be responsible for superconductivity in the quasi-2d high T_c cuprates, the 3d heavy fermion systems and the 2d organic superconductors, and that the character of such spin fluctuations may be very similar and independent of intrinsic dimensionality. Within this SCR treatment, the energy scale of the pairing mechanism is determined predominantly by the energy width of the dynamical spin fluctuations, T_0. For the cases in which the calculations can be compared with existing inelastic neutron scattering measurements of the frequency dependence of the imaginary component of the dynamical susceptibility (eg for the case of $La_{1.85}Sr_{0.15}CuO_4$, Hayden et al, 1996) the agreement is remarkably good. It is particularly interesting to note that SCR theory predicts a linear dependence of T_c upon T_0 as shown in Figure 4. Moreover, the ratio T_c/T_0 (ie approximately 1/30) is of precisely the same order as the ratio T_c/T_B obtained entirely independently for the exotic superconductors in the scaling relations discussed above and shown in Figure 2. This suggests that energetically T_0 is closely similar to T_B. It is therefore worth considering the possibility that the real space local pairing of superconducting carriers necessary for a BE-like condensation could be mediated by such spin fluctuations.

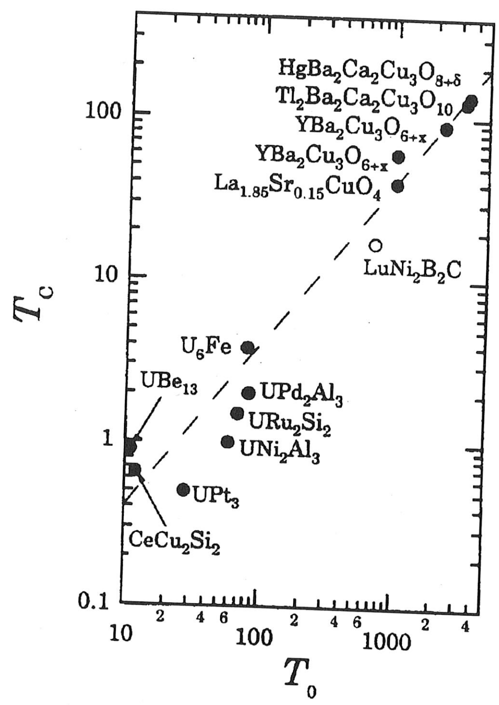

Figure 4. *The SCR prediction for the scaling of superconducting transition temperature with spin fluctuation temperature T_0 (from Nakamura et al, 1996)*

On a different note, a recent theoretical treatment of phase fluctuations in low carrier density superconductors by Emery and Kivelson (1995) is also of some relevance to our discussion of energy scales appropriate to Bose condensation. In Emery and Kivelson's treatment, the stability of the superconducting state to phase fluctuations (which in many ways are analogous to spin waves in the ferromagnetic state) is considered. In estimating the kinetic energy of the phase fluctuations a long range phase ordering temperature, T_θ , is introduced. This characteristic temperature is, in fact, directly analogous to the Bose-Einstein condensation temperature T_B. The calculation of T_θ depends strongly upon the choice of a significant length scale parameter, a, with which to characterize the superconducting ground state. Emery and Kivelson choose the superconducting coherence length ξ to represent this length scale. If instead they had selected the arguably more appropriate inter-particle distance for a, their estimated values of T_θ for many superconductors would be extremely close to the value of T_B determined above, thereby reinforcing our suggestion that T_B is the most appropriate energy scale for pair formation in the exotic superconducting systems.

5 Conclusions

We have shown that μSR techniques can provide information on the superconducting state that goes well beyond the characterisation of internal field distributions and flux lattice morphology. Indeed it appears that μSR can provide the basis for a classification scheme for superconducting systems which may help to discriminate effectively between conventional BCS superconductors and strongly coupled exotic, BE-like superconductors, whilst also highlighting the fundamental similarities between a rather diverse group of exotic superconductors. In this latter respect recent transverse field muon spin rotation measurements on novel superconducting systems, such as the topical nickel borocarbides (Hillier and Cywinski 1997), are also revealing behaviour (such as a ratio of $T_c/T_F \approx 1/100$, independent of doping), which suggests some similarities with the exotic and unconventional superconductors. A systematic study of the many known classes of superconducting compounds using μSR may well provide a firm experimental basis upon which a theoretical interpolation between BCS and BE condensation mechanisms can be developed.

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