Aspects of fundamental muon physics

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1 Introduction

The muon is a second generation fundamental fermion which belongs together with the electron and tauon to the group of charged leptons. To our present knowledge it behaves as a point-like particle which is subject to electromagnetic, weak and gravitational interactions. It differs from the electron and tauon in its behaviour and properties solely through its mass which is called lepton universality. The muon itself and the muonium atom, its bound state with an electron, have been widely used to investigate fundamental laws and symmetries in physics. Since all their known interactions can be calculated very accurately, they have also been employed to determine very accurate values of fundamental constants. New results on the muon magnetic anomaly, the muonium hyperfine structure and a muonium 1s-2s laser excitation are described in some detail as well as a new accurate determination of the fine structure constant. Future possibilities are discussed in view of new accelerator developments.

In particle physics all unambiguous experimental observations to date can be described by a theoretical framework known as the "standard model". In this theory matter in nature is composed of fundamental point-like fermions which are called quarks: up (u), down (d), strange (s), charm (c), top (t) , bottom (b) and leptons: electron (e), electron neutrino (ν_e), muon (μ), muon neutrino (ν_μ), tauon (τ), tauon neutrino (ν_τ) and their respective antiparticles (Figure 1). Two quarks carrying either -1/3 or 2/3 electric charge units and a charged lepton carrying one electric charge unit and a corresponding electrically neutral neutrino form one generation. Three particle generations are known to date. All ordinary matter consists of first generation fundamental fermions (Figure 2). Three different interactions – gravitation, electromagnetic and weak – are experienced by all these particles. A fourth fundamental force – the strong interaction – affects only the quarks (Figure 3). The basic interactions between the fundamental fermions are mediated by gauge bosons – the graviton, the photon, the W- and Z-bosons and eight gluons (Figure 4).

A major step forward in the recognition of the basic laws in physics was the successful description of electricity and magnetism in a single theory by Maxwell in the last century. A second step followed in the last thirty years by unifying electromagnetic and weak interaction in the electroweak standard model by Glashow, Salam and Weinberg. At present there are various attempts to continue the search for a common description of fundamental forces in one single theory. Such efforts are known, e.g. as grand unification theories (GUT's). It should, however, be noted that besides the great successful description of all particle physics, the standard model leaves many open questions about the physical nature of observed processes and the parameters describing particles and interactions. For example, interaction strengths and particle masses as well as the origin for parity violation in weak interaction are put 'by hand' into the theory.

FERMIONS Spin=1/2, 3/2, 5/2, ...

Leptons, Spin=1/2			
	Flavour	Mass [GeV/c^{-2}]	Electric charge [e]
ν_e	electron neutrino	< 7 x 10^{-6}	0
e	electron	0.511	-1
ν_μ	muon neutrino	< 0.2	0
μ	muon	106	-1
ν_τ	tau neutrino	< 30	0
τ	tauon	1777.1	-1

Quarks, Spin=1/2			
	Flavour	approximate Mass [GeV/c^{-2}]	Electric charge [e]
u	up	0.005	2/3
d	down	0.01	-1/3
c	charm	1.5	2/3
s	strange	0.2	-1/3
t	top	173	2/3
b	bottom	4.7	-1/3

Figure 1. *Fundamental fermions of the standard model.*

Examples of bosonic hadrons

Mesons $q\bar{q}$					
Symbol	Name	Quark content	Electric charge	Mass [GeV/c^{-2}]	Spin
π^+	Pion	$u\bar{d}$	+1	0.140	0
K^-	Kaon	$s\bar{u}$	-1	0.494	0
ρ^+	Rho	$u\bar{d}$	+1	0.770	1
D^+	D^+	$c\bar{d}$	+1	1.869	0
η_c	Eta-c	$c\bar{c}$	0	2.979	0

Examples of fermionic hadrons

Baryons qqq and antibaryons $\bar{q}\bar{q}\bar{q}$					
Symbol	Name	Quark content	Electric charge	Mass [GeV/c^{-2}]	Spin
p	Proton	uud	+1	0.938	1/2
$\bar{p}$	Antiproton	$\bar{u}\bar{u}\bar{d}$	-1	0.938	1/2
n	Neutron	udd	0	0.940	1/2
Λ	Lambda	uds	0	1.116	1/2
Ω	Omega	sss	-1	1.672	3/2

Figure 2. *Some hadrons.*

Accurate experiments can stringently test the theoretical description of processes in physics and explore the limits of the physical picture. The standard model of the unified electroweak interaction has been confirmed with an impressive precision in high energy electron-positron collider experiments in the LEP storage ring at CERN (Steinberger 1991). On the other end of the energy scale electroweak theory can be confronted with

Properties of Interactions

Property \ Interaction	Gravitation	Weak	Electromagnetic	Strong: Fundamental	Strong: Rest
		(Electroweak)			
acts on	mass - energy	flavour	electric charge	colour charge	strong "rest" interaction
particles affected	all	quarks, leptons	electrically charged	quarks, gluons	hadrons
interaction mediating particle	graviton (not yet observed)	W^+, W^-, Z^0	γ	gluons	mesons
strength for 2 quarks at: 10^{18} m (relative to electromagn.)	10^{-41}	0.8	1	25	-
strength for 2 quarks at: $3x10^{17}$ m (relative to electromagn.)	10^{-41}	10^{-4}	1	60	-
for 2 protons in nucleus	10^{-36}	10^{-7}	1	-	20

Figure 3. *Properties of fundamental interactions.*

BOSONS Spin = 0, 1, 2, ...

Electroweak Spin = 1	Mass $[GeV/c^{-2}]$	Electric charge [e]
γ Photon	0	0
W^-	80.22	-1
W^+	80.22	+1
Z^0	91.187	0

Strong or Colour Spin = 1	Mass $[GeV/c^{-2}]$	Electric charge [e]
g gluon	0	0

Figure 4. *Fundamental (gauge) bosons of the standard model.*

the material furnished by atomic spectra. Information can be extracted which is complementary to the results from high energy physics (Lanacker *et al.* 1992). It is, for example, a remarkable feature of nature that the weak mixing angle $\sin^2\theta_W$ appears to be constant over 10 orders in the square of the four-momentum transfer Q^2, from the experiments at LEP to the measurements of parity violation in atomic Cesium. Although the effects are small in atomic systems, they can be carefully surveyed with highly precise spectroscopy thanks to modern microwave and laser technology. In general, crucial and fundamental tests require systems where the contribution from well known interactions, particularly the electromagnetic, are well understood. Here single particles like electron, muon and their neutrinos offer excellent opportunities. Among the important experiments are measurements of the parameters such as, for example, the masses and magnetic moments, as well as investigations of decay modes. Next to single particles, one-electron atoms like hydrogen (pe^-) and muonium (μ^+e^-) are the simplest systems available (Figure 5).

In 1932 when a muon was first observed in the cloud chamber of P. Kunze in Rostock, Germany, (Kunze 1933), the discovery remained rather unspectacular, because the standard model of the time had only protons, electrons and photons. The new particle was unforeseen. Years later, after Yukawa had predicted a particle exchange process for the nuclear force, the rediscovery by Anderson and Neddermeyer became more famous, although the muon's identity was mistaken for the particle we now know to be the pion (π). Since then, the muon with its accurately known properties, its decays and its bound states were of crucial importance for model building in basic physics as well as they served as tools in applied sciences.

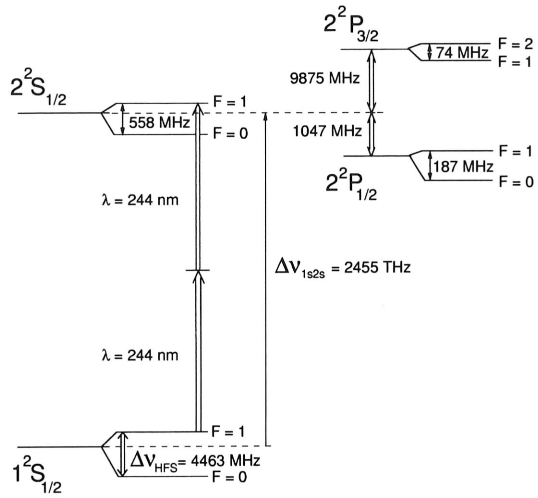

Figure 5. *The energy levels of muonium for principal quantum numbers n =1 and n =2. The level scheme is analogous to atomic hydrogen. The energy scale for gross structure is somewhat smaller due to reduced mass. The hyperfine splitting is larger because of the larger muon magnetic moment compared to the proton. The indicated gross, fine and hyperfine structure transitions have now been studied. The most accurate measurements are the ones involving the n =1 ground state in which the atoms can be produced efficiently.*

Atomic hydrogen has played an important role in the history of modern physics. The successful description of the spectral lines of the hydrogen atom by the Schrödinger equation (Schrödinger 1926) and especially by the Dirac equation (Darwin 1928) had a large impact on the development of quantum mechanics. Precision measurements of the ground state hyperfine structure splitting by Nafe, Nelson and Rabi (Nafe *et al.* 1947, Nafe and Nelson 1948) in the late 1940's were important contributions to the identification of the magnetic anomaly of the electron. Together with the observation of the "classical" $2^2S_{1/2} - 2^2P_{1/2}$ Lamb shift by (Lamb and Retherford 1950) they pushed the development of the modern theory of Quantum Electrodynamics(QED).

Today the anomalous magnetic moment of the electron, which is defined as $a_e = (g_e - 2)/2$, where g_e is the electron g-factor, and the ground state hyperfine structure splitting $\Delta\nu^H_{\mathrm{HFS}}$ of atomic hydrogen are among the most well known quantities in physics. Experiments in Penning traps with single electrons (Van Dyck 1990) have determined a_e=1 159 652 188.4(4.3)$\times 10^{-12}$ to 3.7ppb. QED calculations have reached such a precision that the fine structure constant α can be extracted to 3.8ppb (Kinoshita 1996). Hydrogen hyperfine structure measurements yield $\Delta\nu^H_{\mathrm{HFS}}$ = 1 420.405 751 766 7(9)MHz (0.006ppb) (Essen *et al.* 1971, Hellwig*et al.* 1970) and hydrogen masers even have a large potential as frequency standards. However, the theoretical description of the hyperfine splitting is limited to the ppm level by the fact that the proton's internal structure is not known well enough for calculations of nearly similar accuracy (Sapirstein 1990, Yennie 1992). Neither can experiments supply the necessary data on the mean square charge radius and the polarisability, for example from electron scattering, nor is any theory, for example low energy quantum chromodynamics (QCD), in a position to yield the protons internal charge structure and the dynamical behavior of its charge carrying constituents. The situation is similar for the $2S_{1/2}$- $2P_{1/2}$ Lamb shift in hydrogen, where atomic interferometer experiments find $\Delta\nu^{H}_{2S_{1/2}-2P_{1/2}} = 1\,057.851\,4(19)$MHz (1.8ppm) (Palchikov *et al.* 1985, Karshenboim 1997, van Wijnjaarden *et al.* 1998). The knowledge of the proton's mean square charge radius limits any calculations to the 10ppm level of precision (Sapirstein 1990,Yennie 1992 ,Karshenboim 1996b, Shabaev 1998).

The 1s-2s level separation in atomic hydrogen has reached a value of $\Delta\nu_{1s-2s,H} = 2\,466\,061\,413\,187.34(84)$kHz - a fascinating accuracy for optical spectroscopy – and one can expect even further significant improvements (de Beauvoir *et al.* 1997, Udem *et al.* 1997). The Rydberg constant has been extracted by comparing this transition with others in hydrogen and amounts to $R_\infty = 10\,973\,731.568\,639(91)\text{cm}^{-1}$. It is today's best known fundamental constant. However, the knowledge of the mean square charge radius limits comparison between experiment and theory again at the 40kHz level. In fact, in the case of deuterium the measurements have been employed to infer deuteron parameters (Huber *et al.* 1998).

In addition to the information obtainable from the natural isotopes hydrogen (pe^-), deuterium (de^-) and tritium (te^-) or hydrogen-like ions of natural elements such as $((^Z_AXe^-)^{(Z-1)+})$, exotic hydrogen-like atoms can provide further information about the interactions between the bound particles and the nature of these objects themselves (Jungmann 1994). Such systems can be formed by replacing the electron in a natural atom by an exotic particle (e.g. $\mu^-, \pi^-, K^-, \overline{p}$), or by electron capture of an exotic "nucleus" (e.g. e^+, μ^+, π^+). Even atoms consisting of two exotic particles have been produced occasionally, e.g. $\pi^+\mu^-$ and $\pi^-\mu^+$ were found in in-flight decays of neutral kaons K^0_L (Coombes *et al.* 1977). Some of the systems are compared in Table 1 with respect to the possible spectroscopic resolution for the 1S-2S and the ground state hyperfine structure transition.

Muonic atoms $((\mu^-{}^A_ZX))$ and ions $((\mu^-{}^A_ZX)^{n+})$ are of particular interest for spectroscopy, since the Bohr radius a_μ is about 207 times smaller for muons than for electrons, $a_\mu = (m_e/m_\mu)a_0$, where m_e and m_μ are the masses of the electron and the muon and $a_0 = \hbar^2/(m_e e^2) = 0.529\times10^{-10}$m, with the electric charge unit e and Planck's constant $\hbar$. Bound muonic states are therefore more sensitive to the properties of the nuclei. This has been widely applied for determinations of nuclear charge moments and for examining nuclear polarisation (Schaller 1992, Rosenfelder 1992,Fricke *et al.* 1995). The muon orbit for higher nuclear charges Z is significantly smaller than the electron Compton wavelength $\lambda\!\!\!^{-}_C = \hbar/(m_e c) = 3.86\times10^{-13}$m, which is the typical spatial dimension of vacuum fluctuations. In contrast to electronic systems, the vacuum polarisation contributions in muonic atoms are substantially larger than the self energy, since the Uehling potential, which describes to lowest order the modification of the nuclear potential due to vacuum fluctuations, scales approximately with the cube of the particle mass and the self energy is inversely proportional to it. Higher order vacuum polarisation contributions are needed for a satisfactory description. There is special interest in muonic hydrogen ($p\mu^-$), since, in principle, one could obtain from a measurement of its ground state hyperfine splitting and the 2S-2P Lamb shift more detailed information on the proton's charge radius and polarisability (Jungmann 1992b, Bakalov 1991, Jungmann 1994, Boshier *et al.* 1996, Karshenboim 1997, Taqqu D *et al.* 1998).

The electroweak standard model can be critically tested by a measurement of the four coupling constants describing the parity odd neutral current interaction in the two

	Positronium $e^+ e^-$	Muonium $\mu^+ e^-$	Hydrogen $p\ e^-$	Muonic Helium4 $(\alpha\mu^-)e^-$	Pionium $\pi^+ e^-$	Muonic Hydrogen $p\ \mu^-$
$\Delta\nu_{1S-2S}$ [THz]	$1233.6^\dagger$	2455.6	2466.1	2468.5	2458.6	4.59×10^5
$\delta\nu_{1S-2S}$ [MHz]	$1.28^\dagger$	.145	1.3×10^{-6}	.145	12.2	.176
$\frac{\Delta\nu_{1S-2S}}{\delta\nu_{1S-2S}}$	9.5×10^8	1.7×10^{10}	1.9×10^{15}	1.7×10^{10}	2.0×10^8	2.6×10^{12}
$\Delta\nu_{\mathrm{HFS}}$ [GHz]	203.4	4.463	1.420	4.466	--	4.42×10^7
$\delta\nu_{\mathrm{HFS}}$ [MHz]	1200	.145	4.5×10^{-22}	.145	--	.145
$\frac{\Delta\nu_{\mathrm{HFS}}}{\delta\nu_{\mathrm{HFS}}}$	1.7×10^2	3.1×10^4	3.2×10^{24}	3.1×10^4	--	3.1×10^8

Table 1. *The ground state hyperfine structure splitting $\Delta\nu_{\mathrm{HFS}}$ and the 1s-2s level separation $\Delta\nu_{1S-2S}$ of hydrogen and some exotic hydrogen-like systems offer narrow transitions for studying the interactions in Coulomb bound two-body systems. In the exotic systems the linewidth has a fundamental lower limit given by the finite lifetime of the systems, because of annihilation, as in the case of positronium, or because of weak muon or pion decay. The very high quality factors (transition frequency divided by the natural linewidth $\Delta\nu/\delta\nu$) in hydrogen and other systems with hadronic nuclei can hardly be utilised to test the theory because of the insufficiently known charge distribution and dynamics of the charge carrying constituents within the hadrons. (†Only the 1S-2S splitting in the triplet system of positronium is considered in this table.)*

isotopes of muonic boron (Lanacker *et al.* 1992, Bernabeu 1992, Missimer and Simons 1990). The existence of additional neutral vector bosons and leptoquarks and composite quark contact interaction could be surveyed with a sensitivity beyond the possibilities of LEP. The 2S and 2P states are mixed by the weak interaction. An observable for a future measurement could consist of directional correlation between the muon decay electron and a single photon emitted from the 2S-1S transition.

The muonic helium atom $((\alpha\mu^-)^+e^-)$ (Souder *et al.* 1980, Orth *et al.* 1980, Gardener *et al.* 1982) consists of a pseudo-nucleus $(\alpha\mu^-)^+$, which is itself a hydrogen-like system, and an electron. It is the simplest atomic system involving both a negative muon and an electron. A precise value for the magnetic moment of the negative muon has been deduced from the Zeeman splitting of the ground state hyperfine structure as a test of the CPT theorem, which is the basic assumption that all physics remains the same, if the charges, the parity and the time direction (described by their operators C,P and T) involved in a process under consideration is reversed in its descriptions.

Already in the 1970's the muonic helium ion $(\alpha\mu^-)^+$ was the first exotic atomic system for which a successful laser experiment has been reported (Carboni *et al.* 1976,Carboni *et al.* 1977). Allowed electric dipole transitions between the n =2 fine structure levels have been induced. A precise test of QED vacuum polarisation could be established and the rms charge radius of the α-particle was extracted. The experiment needed as a prerequisite the metastability of the 2S state at high pressures ($\approx$40 bar), which could not be confirmed in several independent approaches (von Arb *et al.* 1984, Eckhause *et al.* 1986, Rosenkranz *et al.* 1990) and a second attempt of a laser experiment could not confirm the existence of a signal (Hauser *et al.* 1992), which leaves us with an open puzzle.

In purely hadronic systems like pionic hydrogen ($p\pi^-$) (Chatellard *et al.*, 1997) and antiprotonic hydrogen (Beer *et al.* 1991), called protonium ($p\bar{p}$) (Auld *et al.* 1978, Klempt 1989), the transition frequencies are dominantly due to the Coulomb interaction. They bear additional line shifts and broadenings in the spectral lines due to strong interaction between the constituents. They offer the possibility to study the strong interaction between the particles at zero energy. From the transition frequencies in pionic atoms (or possibly in far future from the pionium atom (π^+e^-) (Mundinger *et al.* 1989) one can derive a value for the pion mass (Jeckelmann *et al.* 1986). At present the determination of the best upper limit for the muon neutrino mass from the muon momentum from pion decays at rest (Daum *et al.* 1991,Daum *et al.* 1992, Assamagan *et al.* 1994) is spoiled by the uncertainties of the pion mass.

For leptons no internal structure is known so far. Scattering experiments have established that electron (e), muon(μ) and tauon (τ) behave like point-like particles down to dimensions of less than 10^{-18}m (Martyn *et al.* 1990, Kinoshita and Marciano 1990) which is three orders of magnitude below the proton's rms charge radius (Sick 1982, Simon *et al.* 1980, Hand *et al.*1963). Purely leptonic hydrogen-like systems like positronium (e^+e^-) (Deutsch 1951, Mills and Chu 1990) and muonium (μ^+e^-) (Hughes *et al.*1960, Hughes and zu Putlitz 1990, Hughes 1997) have interesting perspectives for testing bound state QED, for searching for deviations from present models and for testing fundamental symmetry laws.

Positronium is a particle anti-particle system and significantly differs from muonium and hydrogen-like systems with different constituent masses. The Furry picture, where the electronic states are in zeroth order solutions of the Dirac equation in an external electrostatic field and which is successfully applied for the heavier systems, is not appropriate. A theoretical description must start from the fully relativistic Bethe-Salpeter formalism or an approximation to it (Mohr 1988). Depending on the C-parity of the state, the ground state of the positronium atom annihilates into two (1^1S_0) or three (1^3S_0) photons. Standard theory was confirmed in various searches for rare decay modes which have been carried out (Mills and Chu 1990) in order to find unknown light particles, e.g. axions, or violations of fundamental laws, e.g. C-parity symmetry. For the ^3S-states annihilation into a single virtual photon causes significant shifts at the fine structure level. Microwave spectroscopy experiments on fine structure transitions in the n =2 state of the triplet system (parallel e^- and e^+ spin) are in moderate agreement with each other and with theoretical calculations (Pachucki and Karshenboim 1998). The ground state hyperfine splitting presents a theoretical challenge and would call for more precise measurements (Pachucki 1998).

Positronium was the second exotic system in which laser excitation could be achieved (Chu *et al.* 1984). The 1S-2S transition frequency has been measured to be $\Delta\nu^{PS}_{1S-2S}$(expt) = 1 233 607 216.4(3.2)MHz (2.6ppb) (Fee *et al.* 1993), which agrees with $\Delta\nu^{PS}_{1S-2S}$(theory)

= 1 233 607 221.0(1.0)MHz (0.8ppb) (Pachucki and Karshenboim 1998), within two standard deviations. This constitutes a test of the QED contributions to ≈10ppm. The theoretical uncertainty of 1 MHz is the estimated size of uncalculated higher order (higher than $\alpha^4 R_\infty$) terms (Fell 1992). Their evaluation will be cumbersome, because of the inflation of the number of Feynman diagrams due to virtual annihilation. From the result one can conclude that electron and positron masses are equal to 2ppb, the best test of mass equality for particle and antiparticle next to the $K^0\overline{K^0}$ system (Caso *et al.* 1998), where the relative mass difference is $\leq 10^{-18}$.

Precision experiments on muonium offer a unique opportunity to investigate bound state QED without complications arising from nuclear structure and to test the behavior of the muon as a heavy leptonic particle and hence the electron-muon(-tau) universality, which is fundamentally assumed in QED theory. Of particular interest is the ground state hyperfine structure splitting, where experiment (Mariam *et al.* 1982) and theory (Sapirstein and Yennie 1990, Yennie 1992) agree at 300ppb which is at a higher level of precision than in the case of atomic hydrogen, where the limitation in theory arises from the knowledge of the muon mass. An accurate value for the fine structure constant α can be extracted to 140ppb. The Zeeman effect of the ground state hyperfine sublevels yields the most precise value for muon the magnetic moment μ_μ with an accuracy of 360ppb. Preliminary results from a new experiment will be discussed in Section 5.1.1 which should improve these figures by about a factor of three. Signals from the "classical" $2^2S_{1/2}$-$2^2P_{1/2}$ Lamb shift in muonium have been observed (Oram *et al.* 1984, Badertscher *et al.* 1984). However, with 1.4% precision they are not yet in a region where they can be confronted with theory. The sensitivity to QED corrections is highest for the 1S-2S interval in muonium due to the approximate $1/n^3$ scaling of the Lamb shift. Compared to hydrogen, the radiative recoil and the relativistic recoil effects are larger by a factor of m_p/m_μ ≈8.9, where m_p is the proton mass and m_μ the muon mass. A hydrogen-muonium isotope shift measurement in this transition as well as a technically only slightly more difficult measurement of the 1s-2s transition frequency in muonium can lead to a new and accurate figure for the muon mass m_μ. The transition has recently been excited successfully in two independent experiments (Chu *et al.* 1988, Danzmann *et al.* 1989, Jungmann *et al.* 1991, Maas *et al.*1994).

In a series of three (g-2) experiments at CERN the magnetic anomaly a_μ of the free muon itself has been carefully studied (Prigl *et al.* 1998, Farley and Picasso 1990) by observing the difference (ω_a)of the spin precession frequency and the cyclotron frequency in the homogeneous magnetic field of a storage ring. The relative contributions from heavier virtual particles to the magnetic anomaly are larger for the muon by a factor of $\sim (m_\mu/m_e)^2) \approx 4.2\times10^4$ compared to the electron. This makes the muon a better probe for new physics. A new muon storage ring experiment at BNL, Brookhaven, USA, aims for a precision of 0.35ppm for a_μ (Roberts 1992). The production of virtual particles can be explored into the TeV/c^2 mass range and tighter constraints for new physics beyond the standard model(Kinoshita and Marciano 1990), will be set.

The sensitivity is similar to that of experiments at the present and future high energy machines (LEP 200, Fermilab $p\overline{p}$. LHC) and partly beyond the scope of those. The experiment will test the renormalisability of the weak interaction in a cleaner way than from the masses of Z- and W-bosons, where the effect is clouded by radiative corrections involving top quarks and Higgs bosons, or from nuclear β-decays, where complications arise from nuclear structure. In order to reach the desired goal, the muon mass must be known to at least ~0.1ppm. At present, muonium spectroscopy is the only viable way to obtain such a precision.

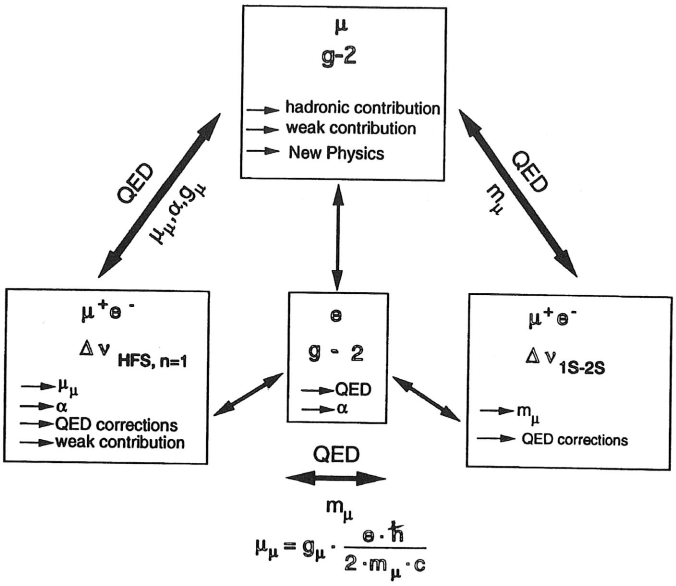

Figure 6. *Relation between measurements of the muon and electron anomalous magnetic moments and spectroscopy of the energy levels in muonium. Each experiment needs precise results obtained in the other ones. The whole set is a stringent consistency test for theory and the set of fundamental constants involved. The only external input required is the hadronic correction to* a_μ.

The spectroscopic experiments in muonium are closely inter-related with the determination of the muon's magnetic anomaly a_μ through the relation $\mu_\mu = (1 + a_\mu)e\hbar/(2m_\mu c)$. The results from all experiments establish a self consistency requirement for QED and electroweak theory and the set of fundamental constants involved (Figure 6) The constants α, m_μ, μ_μ are the most stringently tested important parameters. The only necessary external input are the hadronic corrections to a_μ which can be obtained from a measurement of the ratio of cross sections $(e^+e^- \to \mu^+\mu^-) \,/\, (e^+e^- \to \text{hadrons})$. Although, in principle, the system could provide the relevant electroweak constants, the Fermi coupling constant G_F and $\sin^2\theta_W$, the use of more accurate values from independent measurements may be chosen for higher sensitivity to new physics. As a matter of fact, an improvement upon the present knowledge of the muon mass at the 0.35ppm level is very important for the success of a new measurement of the muon magnetic anomaly presently under way at the Brookhaven National Laboratory. The relevance of the experiment, which aims for 0.35ppm accuracy, arises from its sensitivity to contributions from physics beyond the standard model and the clean test it promises for the renormalisability of electroweak interaction (Prigl *et al.* 1998, Farley and Picasso 1990, Roberts 1992).

The decay of the muon itself is an interesting process to study. From precision measurements of its lifetime one can extract the best value for the important Fermi coupling constant describing the coupling strength in weak interaction. Also the muon analogon to inverse β-decay, the muon capture process can reveal insights in fundamental symmetries, for example in the possiblity of an asymmetry in nature against reversing the time direction in the theoretical description of a process (Deutsch 1992). Rare decays of muons and decays forbidden by lepton number conservation have been studied very carefully since the late 1940's in order to find hints on the muon's nature. Today there are various attempts to identify in forbidden muon decays physics beyond the standard model. Of interest in this connection are searches for the decay mode $\mu \to e\gamma$ and for μ-e conversion in a muon-nucleon reaction. Particularly the muonium atom, which consists of two leptons from two different lepton generations, offers important possibilities. Since in the bound state the particles are close to each other for relatively long interaction times compared to scattering experiments, one can expect the system to be very sensitive to exotic interactions between the electron and muon. A possible muonium to antimuonium conversion has first been considered by Pontecorvo (1958) in analogy to the $K^0\overline{K^0}$ oscillations. The process violates the separate additive conservation of lepton flavour numbers. Although not provided in the standard model, it would be allowed in many extensions to it like left-right symmetry, supersymmetry and technicolour (Vergados 1986).

2 Magnetic anomaly of the muon

Trapping of elementary particles in combined magnetic and electric fields has been very successfully applied for obtaining properties of the respective species and for determining most accurate values of fundamental constants. The magnetic anomaly of fermions $a = \frac{1}{2}(g-2)$ describes the deviation of their magnetic g-factor from the value 2 predicted in the Dirac theory. It could be determined for electrons and positrons in Penning traps by Dehmelt and his coworkers to 3.7ppb (Van Dyck 1990). Accurate calculations involving almost exclusively the "pure" Quantum Electrodynamics (QED) of electron, positron and photon fields allow the most precise determination of the fine structure constant α (Kinoshita 1996) by comparing experiment and theory in which α appears as an expansion coefficient. The high accuracy to which calculations in the framework of QED can be performed is demonstrated by the satisfactory agreement between this value of α and the ones obtained in measurements based on the quantum Hall effect (Jeffrey *et al.* 1997) as well as the ac-Josephson effect and the gyromagnetic ratio of protons in water (Williams *et al.* 1989), or the number extracted from the very precisely known Rydberg constant (Udem *et al.* 1998) using an accurate determination of the neutron de Broglie wavelength (Krüger *et al.* 1997, Nistler 1998) and well known relevant mass ratios (Caso 1998).

The anomalous magnetic moment of the muon a_μ has a $(m_\mu/m_e)^2 \approx 4\times10^4$ times higher sensitivity to heavier particles and other than electromagnetic interactions. Those can be investigated carefully, as very high confidence in the validity of calculations of the dominating QED contribution arises from the success of QED describing this quantity for the electron.

For the muon a_μ has been measured in a series of three experiments at CERN (Farley and Picasso 1990,) all using magnetic muon storage. Contributions arising from strong interaction amount to 60ppm and could be identified in the last of these measurements.

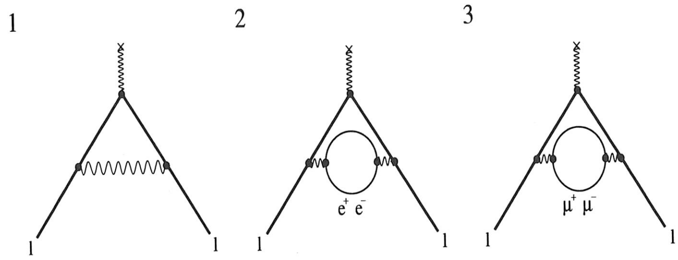

Figure 7. *QED Feynman diagrams of lowest order contributing to muon g-2. They are very similar to the ones responsible for the lowest order Lamb shift in atomic hydrogen.*

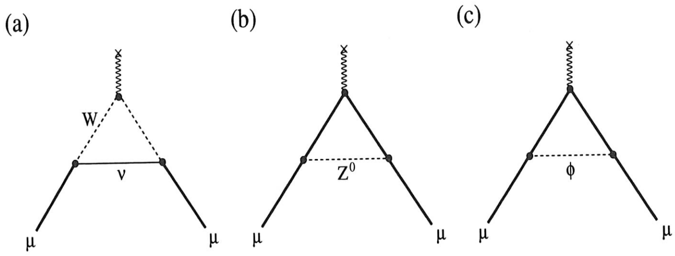

Figure 8. *Lowest order weak processes contributing to muon g-2.*

At the Brookhaven National Laboratory thr new dedicated experiment has started measurements to determine the muon's magnetic anomaly. It aims for 0.35ppm relative accuracy meaning a 20 fold improvement over previous results. At this level it will be particularly sensitive to contributions arising from weak interaction through loop diagrams involving W and Z bosons (1.3ppm). The experiment promises further a clean test of renormalisation in weak interaction. The muon magnetic anomaly may also contain contributions from new physics (Mery*et al.* 1990 Lopez *et al.*1994, Renard *et al.* 1997). A variety of speculative theories can be tested which try to extend the present Standard Model in order to explain some of its not yet understood features. This includes muon substructure, new gauge bosons, supersymmetry, an anomalous magnetic moment of the W boson and leptoquarks. Here this measurement is complementary to searches carried out in the framework of other high energy experiments. In some cases the sensitivity is

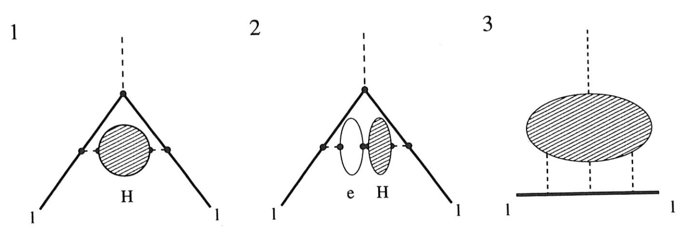

Figure 9. *Strong interaction diagrams contributing to muon g-2. Whereas the processes 1 and 2 can be obtained from measurements of the cross section* $e^+ + e^- \rightarrow$ *anyhadrons, as a function of center of mass energy, process 3 needs to be calculated and causes right now one of the largest uncertainties of the hadronic contribution to muon g-2.*

new physics	sensitivity			other experiments
Muon substructure	Λ	$\geq$	5TeV	LHC similar
excited muon	m_{μ^*}	$\geq$	400GeV	LEP II similar
$W^{\pm}$-boson substructure	Λ	$\geq$	400GeV	LEP II ~100-200GeV
$W^{\pm}$ anomalous magnetic moment	a_W	$\geq$	0.02	LEP II ~0.05, LHC ~0.2
Supersymmetry	$m_{\widetilde{W}}$	$\leq$	130GeV	Fermilab $p\bar{p}$ similar
right handed $W^{\pm}_R$-bosons	$m_{W'}$	$\leq$	250GeV	Fermilab $p\bar{p}$ similar
heavy Higgs boson	m_H	$\leq$	500GeV	
Muon electric dipole moment	D_μ	$\leq$	$4\times10^{-20}ecm$	

Table 2. *Sensitivity to new physics of the g-2 experiment at BNL aiming for 0.35ppm relative accuracy. ([a]for substructure $\Delta a_\mu \sim m_\mu^2/\Lambda^2$.)*

even higher (Table 2).

In the BNL experiment (BNL proposal 1994) polarised muons are stored in a magnetic storage ring of highly homogeneous field B and with weak electrostatic focussing using quadrupole electrodes around the storage volume. The difference frequency of the spin precession and the cyclotron frequencies,

$$\omega_a = a_\mu \frac{e}{m_\mu c} B, \tag{1}$$

is measured, with m_μ the muon mass and c the speed of light, by observing electrons or positrons from the weak decay $\mu^\pm \to e^\pm + 2\nu$. For relativistic muons the influence of a static electric field vanishes (Telegdi 1959), if $a_\mu = 1/(\gamma_\mu^2 - 1)$ which corresponds to $\gamma_\mu = 29.3$ and a muon momentum of 3.094GeV/c, where $\gamma_\mu = 1/\sqrt{1-(v_\mu/c)^2}$ and v_μ is the muon velocity. The momentum needs to be accurate at the 10^{-4} level for a corresponding correction to be below the desired accuracy for a_μ. For a homogeneous field the magnet must have iron flux return and shielding. To meet this, the particular momentum requirement and to avoid magnetic saturation of the iron a device of 7m radius was built. It has a C-shaped iron yoke cross section with the open side facing towards the center of the ring. It provides 1.4513T field in a 18cm gap. The magnet is energised by 4 superconducting coils carrying 5177A current. The storage volume inside of a Al vacuum tank has 9cm diameter.

The magnetic field is measured by a newly developed narrow band magnetometer system which is based on pulsed nuclear magnetic resonance (NMR) of protons in water. It has the capability to measure the absolute field value to ≈50ppb (Prigl *et al.*, 1996). The field and its homogeneity are continuously monitored by 366 NMR probes which are embedded in the Al vacuum tank and distributed around the ring. Inside the storage volume a trolley carrying 17 NMR probes arranged to measure the dipole field and several important multipole components as well a fully computerised magnetometer built from all non-ferromagnetic components is used to map the field at regular intervals. The accuracy is derived from and related to a precision measurement of the proton gyromagnetic ratio in a spherical water sample (Phillips *et al.* 1977). The field homogeneity at present is about 25ppm. It will be improved to the ppm level using mechanical shimming methods and a set of electrical shim coils. The field integral in the storage region is known at present

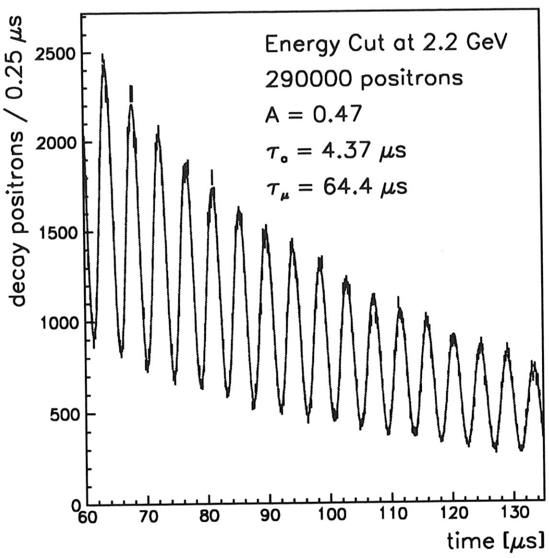

Figure 10. *The observed muon spin precession signal in the new muon g-2 experiment at Brookhaven.*

to better than 1ppm in absolute terms at any time and field drifts of a few ppm/hour were observed. Due to thermal insulation for the magnet yoke and active feedback from the readings of the fixed NMR probes the field path integral can be kept constant within 0.1ppm.

The weak focusing is provided by electrostatic quadrupole field electrodes with 10cm separation between opposite plates. They cover four 39° sections of the ring. The electric field is applied by pulsing a voltage of $\pm$24.5kV for a few ms duration to avoid trapping of electrons and electrical breakdown (Flegel and Krienen 1973).

Due to parity violation in the weak muon decay process the positrons are emitted preferentially into the muon spin direction causing a time dependent variation of the spatial distribution of decay particles in the muon eigensystem which translates into a time dependent variation of the energy distribution observed by detectors fixed inside the ring.

The improvements over previous experiments include an azimuthally symmetric iron construction for the magnet with superconducting coils, a larger gap and higher homogeneity of the field, an electron/positron detector system covering a larger solid angle and using segmented detectors and improved electronics. A major advantage is the two orders of magnitude higher primary proton intensity available at the AGS Booster at BNL. A new key feature will be the direct injection of muons into the storage volume using an electromagnetic kicker as compared to filling the ring with decay muons from injected pions which has been employed so far.

In order for the new muon g-2 experiment to reach its design accuracy besides the field also the muon mass respectively its magnetic moment needs to be known to 0.1ppm or better. An improvement beyond the present 0.36ppm accuracy of this constant can be expected from both microwave spectroscopy of the muonium atom's (μ^+e^-) hyperfine

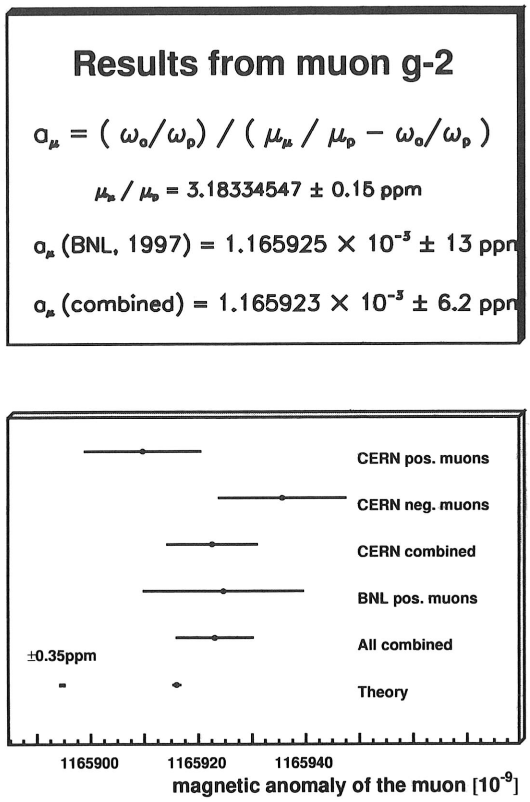

Figure 11. *Results from the startup run of the new muon g-2 experiment.*

structure and from laser spectroscopy of the muonium 1s-2s transition (Jungmann 1994).

Provided the systematic uncertainties, which mainly arise from the knowledge of the convolution of the muon distribution in the storage volume and the magnetic field there as well as from imperfections of the electronics and muon losses, can be controlled at the 10^{-7} level, only the statistical error remains of concern. It is given for the asymmetry frequency ω_a by

$$\frac{\delta\omega_a}{\omega_a} = \frac{\sqrt{2}}{\omega_a A \gamma \tau_\mu \sqrt{N_e}} \tag{2}$$

where A is the observed decay asymmetry and N_e the recorded number of decays. Therefore besides the number of particles in particular a large decay asymmetry is desired.

The experiment achieved in its startup run 1997 a measurement of value for the muon magnetic anomaly to 13ppm. It is in agreement with previous CERN results and with theory. The near future will show whether there is new physics to be expected, when the statistical uncertainty can be significantly lowered.

The main means to achieve this, the electromagnetic magnetic kicking device, could be shown in summer 1998 to allow muon injection yields of more than one order of magnitude larger than with pion injection.

According to the standard theory an elementary particle is not allowed to have a finite permanent electric dipole moment (edm) as this would violate time reversal symmetry. An edm of the muon would manifest itself in the Brookhaven g-2 experiment in a time dependent up down asymmetry of decay positrons which can be searched for along with the muon g-2 measurements (Farley and Picasso 1990). The new g-2 experiment will be sensitive to yield here a 10 fold improvement over the present limit at $3.7(3.4)\times10^{-19}$ e cm. A very promising approach seems to be the proposed much more sensitive search for a muon edm in a dedicated succeeding experiment (Semertzidis *et al.* 1997). Major modifications of the present g-2 magnet setup would be required which involve the application of a radial electric field to compensate g-2 precession and the switching from electrostatic to alternating gradient focusing by replacing the pole tips of the magnet with appropriately shaped pieces of iron. In case of a finite electric dipole moment a time dependent asymmetry in the muon decay rates counted above and below the storage region is expected as a signature. Such an experiment may achieve up to four orders of magnitude improvement and could reach a level of sensitivity at which several theoretical models, particularly those involving supersymmetry, could be tested. For some models the sensitivity to new physics would be higher than for edm searches of the electron or the neutron, particularly if the effect would scale nonlinear with the mass of the leptons.

3 Muon decays

3.1 Allowed muon decays

Muons can originate from pion (π) decays. This process is dominant in the production of cosmic ray muons after pion production through the bombardment of nuclei in the gas molecules of the atmosphere by energetic protons of cosmic origin. Cosmic muon fluxes at sea level are about 180 particles/(m^2s). Pion creation in nuclear reactions with proton beams is also employed in todays meson factories — laboratories which muon fluxes of up to a few times 10^6/sec can be obtained. The positive (negative) muon itself decays mostly into a (positron) electron and two neutrinos.

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \quad , \quad \mu^+ \rightarrow e^+ + \nu_e + \overline{\nu_\mu}$$

$$\pi^- \rightarrow \mu^- + \overline{\nu_\mu} \quad , \quad \mu^- \rightarrow e^- + \overline{\nu_e} + \nu_\mu .$$

These equations reflect the fact that the negative muon is a particle, the positive muon an antiparticle as antineutrinos are indicted by overlining. Both pion and muon decays are due to the weak interaction and violate parity-conservation (Garwin *et al.* 1957,Friedman and Telegdi 1957). The non-conservation of parity in pion decay causes the spin of the positive (negative) decay muon to be directed against (in) its propagation direction, because neutrinos only have negative helicity. In a positive (negative) muon decay the positrons (electrons) are preferentially emitted in (against) the direction of the muon spin. This effect is widely used experimentally to determine the average spin of a muon ensemble after their decay. The maximum kinetic energy of the decay positrons or electrons is $E_{\text{max}} = 52.3$MeV corresponding to half the muon mass and the assumption of negligible muon neutrino mass.

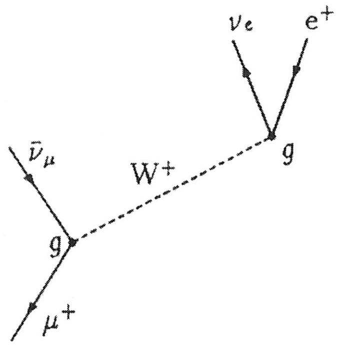

Figure 12. *Feynman diagram describing the weak process of muon decay.*

The probability with which the positron is emitted in a certain direction within a certain energy range can be calculated by integrating over the neutrinos' momenta, because they usually cannot be observed. The double differential distribution $W(\theta, \epsilon)$ of the decay positrons with respect to the spin of the muon is given by

$$\begin{aligned} d^2W &= W(\theta,\epsilon)\, d\epsilon\, d(\cos\theta) \\ &= \frac{G^2 m_\mu^5}{192\pi^3} \times (3-2\epsilon)\,\epsilon^2 \left[1 + \frac{(2\epsilon-1)}{3-2\epsilon}\cos\,\theta\right] d\epsilon\, d(\cos\theta), \end{aligned} \tag{3}$$

where $\epsilon = E/E_{\rm max}$ and θ is the angle between the muon spin and the momentum of the emerging positron. The asymmetry in the angular distribution is a consequence of parity violation in the weak interaction. The energy spectrum of the positrons or electrons follows from Equation (3) by only integrating over $\cos\theta$

$$dW(\epsilon) = W(\epsilon)d\epsilon = \left[2(3-2\epsilon)\epsilon^2/\tau_\mu\right]\, d\epsilon, \tag{4}$$

with the muon lifetime

$$\tau_\mu = \frac{G^2 m_\mu^5}{192\pi^3}, \tag{5}$$

the experimental value of which is $\tau_\mu = 2.19703(4)\times10^{-6}$s (Caso *et al.* 1998).

The asymmetry factor preceding the cosine function in Equation (4) depends on the positron energy and for an ensemble of muons with polarisation P the asymmetry is

$$a(\epsilon) = \frac{2\epsilon - 1}{3 - 2\epsilon}\times P \quad . \tag{6}$$

For decay positrons in the same plane as the muon spin gives

$$\frac{dN_{e+}}{d\theta} \sim (1 + a\cos\theta) \tag{7}$$

and for the decay electrons in the μ^- decay

$$\frac{dN_{e^-}}{d\theta} \sim (1 - a\cos\theta). \tag{8}$$

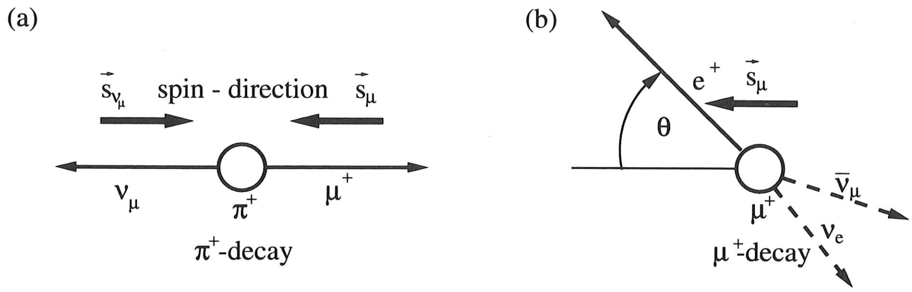

Figure 13. *a) pion decay in the pion rest frame, the spin directions $\vec{s}$ are indicated. b) muon decay in its rest frame.*

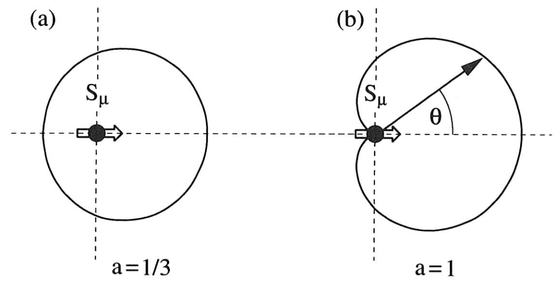

Figure 14. *Muon decay. The rate of the decay positrons is given in a polar diagram with an angle θ with respect to the muon spin. (a) Averaged over all positron energies the asymmetry parameter a is 1/3. For negative muons a is $-1/3$ and the electron is emitted predominantly against the direction of the muon spin. (b) For the highest energy positrons a equals 1.*

integrated over all energies $a = 1/3$. For the highest energies $a = 1$ (see figure 14).

The spectral shape of the muon decay energy spectrum reflects the V-A nature of the decay process. Using a parametrisation of Michel other interaction types can be excluded in precision experiments determining the so called Michel parameters (Scheck 1978, Caso 1998).

A 1.4(0.4)% fraction of all muons decays with the additional release of a photon according to

$$\mu^+ \to e^+ + \nu_e + \overline{\nu_\mu} + \gamma \, .$$

Due to a radiative correction process involving an intermediate photon which vanishes into a electron positron pair, there is a rare decay mode

$$\mu^+ \to e^+ + \nu_e + \overline{\nu_\mu} + e^+ + e^-,$$

which has a $3.4(0.4)\times10^{-5}$ branching ratio (Caso 1998).

Through Equation (5) a strong relation between τ_μ, m_μ and G_F is established. Since radiative corrections due to QED effects to this equation are known well, a precise measurement can yield an accurate value of the G_F, which is one of the most fundamental

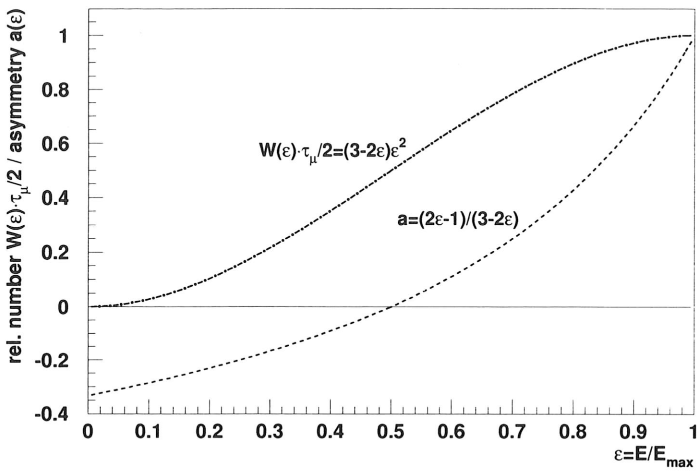

Figure 15. *The relative number $W(\epsilon)$ and the asymmetry a of the spatial distribution versus the energy of the decay positrons. For low energy positrons the asymmetry is negative.*

constants in the electroweak model. Improvements are desired, particularly since other important weak constants like the Z boson mass are precisely known from experiments at LEP, for example. There are at present three efforts at BNL, RAL and PSI to improve τ_μ to the 1ppm level (Navarria *et al.* 1998, Nagamine *et al.*1998, Carey *et al.* 1998).

3.2 Lepton number violating muon decays

To date experiments indicate the conservation of lepton quantum numbers. This fact can be described in several different solely empirical laws (Zeldovitch 1952, Pontecorvo 1959, Cabbibo and Gatto 1960, Konopinski and Mahmoud 1953, Feinberg and Weinberg 961,,Cabbibo 1961), some of which follow additive and some obey multiplicative, parity-like, schemes. Experiments have given no indication yet for favouring any of them. The standard model states for every lepton flavour a separate additively conserved quantum number. However, such lepton numbers have no status, unless their conservation can be associated with a local gauge invariance (Halprin and Masiero1993). Mixings between different generations are well known in the quark sector and the Cabbibo-Kobayashi-Maskawa matrix (Kobayashi and Maskawa 1973), relates the weak quark eigenstates with their mass eigenstates. A familiar example are the K^0-$\overline{K^0}$ oscillations. At present we are left puzzled why leptons do not show any similar mixing. Recent experimental hints for neutrino oscillations, which have a potential for changing this situation, are not covered here (see e.g. Stone 1998).

Many extensions to the standard model have been proposed and are presently discussed which try to explain further some of its not well understood features like e.g. parity violation in weak interaction or particle mass spectra. They are put by hand into this remarkable theoretical framework which appears to serve as an extremely robust description of all confirmed particle physics. Lepton flavor violation (LFV) appears naturally in such models which include Left-Right-Symmetry, Supersymmetry, Technicolor, Grand Unification, String Theories, Compositeness, and many others. Such models continue to stimulate experimental searches in a large range of energies. With some low

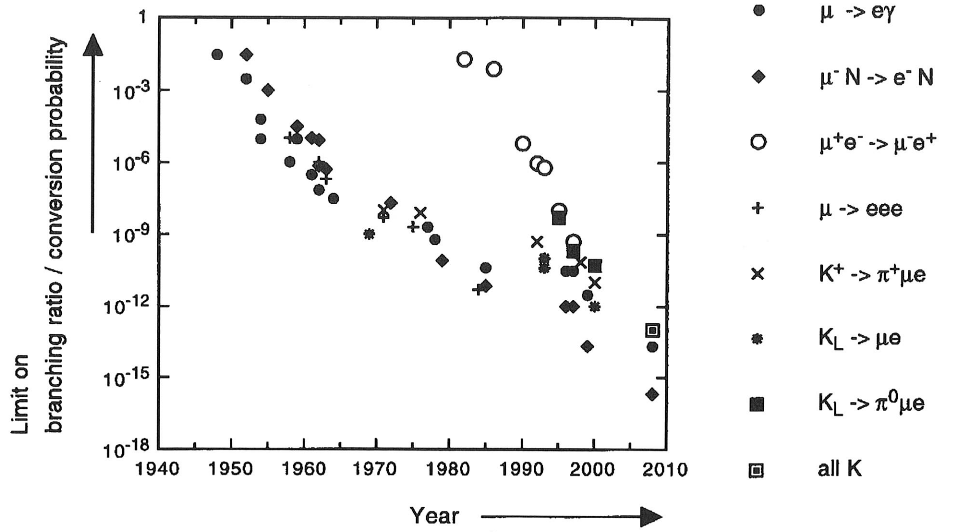

Figure 16. *Dedicated searches for lepton number violating processes involving muons* (μ) *and kaons (K). Recent K experiments and* μ^+e^-–μ^-e^+ *conversion exhibit the most significant gains in sensitivity. Points in 1998 and beyond are projections of possibilities by the respective experimenters.*

energy experiments new physics can be probed at mass scales far beyond the reach of present accelerators or those planned for the future.

Highest sensitivity has generally been reached in dedicated search experiments particularly on Kaons (K) and muons (μ) (Table 3), where also a high discovery potential for new physics exists (Mohapatra 1993), as well as in non accelerator experiments searching for neutrino-less double β-decay. The decays of heavier objects created in high energy collisions, however, can be observed less accurately. The progress in the K and μ field is indicated in Figure 16 which shows more than 10 decades of improvement since the first experiments in the late 1940's. The highest recent gain in sensitivity is for muonium ($\mathrm{M}=\mu^+e^-$) to antimuonium ($\overline{\mathrm{M}}=\mu^-e^+$) conversion due to a new, yet unused signature (see Section 3.5).

The decay $\mu \to e\gamma$ was the first to be searched for shortly after the muon's nature as a heavy electron-like particle became apparent. Searches for rare and forbidden muon decays have been among the most precise experiments in physics since and have always been of special interest in the context of unified gauge theories, as they can provide accurate tests of speculative models and because of the achievable experimental precision they may be able to discriminate between these (Vergados *et al.* 1994). Recently forbidden muon decays have attracted attention, when their possible sensitivity to effects arising in minimal supersymmetry (SUSY) were discussed in theoretical studies (Barbieri *et al.* 1995). It was pointed out that for values of $\tan\beta$ (the ratio of the vacuum expectation

decay			limit	
Z^0	$\rightarrow$	μe	2.5×10^{-6}	(Bugge *et al.* 1996)
Z^0	$\rightarrow$	τe	7.3×10^{-6}	(Bugge *et al.* 1996)
Z^0	$\rightarrow$	$\tau\mu$	1.0×10^{-5}	(Bugge *et al.* 1996)
D^0	$\rightarrow$	μe	1.9×10^{-5}	(Freyberger *et al.* 1996)
D^0	$\rightarrow$	$\pi^0\mu e$	8.6×10^{-5}	(Freyberger *et al.* 1996)
D^0	$\rightarrow$	$\Phi\mu e$	3.4×10^{-5}	(Freyberger *et al.* 1996)
B^0	$\rightarrow$	μe	5.9×10^{-6}	(Ammar *et al.* 1994)
B^0	$\rightarrow$	τe	5.3×10^{-4}	(Ammar *et al.* 1994)
B^0	$\rightarrow$	$\tau\mu$	8.3×10^{-4}	(Ammar *et al.* 1994)
B^0	$\rightarrow$	K μe	1.8×10^{-5}	(Ammar *et al.* 1994)
τ	$\rightarrow$	$e\gamma$	2.7×10^{-6}	(Edwards *et al.* 1997)
τ	$\rightarrow$	$\mu\gamma$	3.0×10^{-6}	(Edwards *et al.* 1997)
K_L	$\rightarrow$	μe	2×10^{-11}	(Molzon 1998)
K_L	$\rightarrow$	$\pi^0\mu e$	3.2×10^{-9}	(Belz 1998)
K^+	$\rightarrow$	$\pi^+\mu e$	4×10^{-11}	(Zeller 1998)
μ^+	$\rightarrow$	$e^+\nu_\mu\overline{\nu}_e$	2.5×10^{-3}	(Eitel 1995)
μ	$\rightarrow$	eee	1×10^{-12}	(Bertl *et al.* 1985)
μ	$\rightarrow$	$e\gamma$	3.8×10^{-11}	(Cooper *et al.* 1997)
μ^-Ti	$\rightarrow$	e^-Ti	6.1×10^{-13}	(Eggli *et al.*1998)
μ^-Ti	$\rightarrow$	e^+Ca	1.7×10^{-12}	(Kaulard J *et al.* 1998)
μ^+e^-	$\rightarrow$	μ^-e^+	$G_{\mathrm{M\overline{M}}} < 3\times10^{-3}G_F$	(Meyer *et al.* 1997)
^{76}Ge	$\rightarrow$	^{76}Se e^-e^-	$T_{1/2} > 1.2\times10^{25}y$	(Klapdor and Hirsch 1997)
			$m_{\nu_e}(Maj.) < 0.45eV$	(Klapdor and Hirsch 1997)

Table 3. *Recent upper limits on lepton number violating processes (90% C.L.).*

values of the two Higgs fields involved) which exceed about 3, the branching ratio should be above $\approx 10^{-14}$ for a decay $\mu \rightarrow e\gamma$ and above $\approx 10^{-16}$ for $\mu \rightarrow e$ conversion on a Ti nucleus, almost independent of all other parameters in the model. This has stimulated a letter of intent to the Paul Scherrer Institute (PSI), Switzerland, and a proposal to BNL to search for the respective processes.

In the field of searching for SUSY effects in low energy experiments rare decay experiments are in some competition with the just started new precision measurement (Miller *et al.* 1997) of the muon magnetic anomaly a_μ where the contribution from SUSY is of order $a_\mu(SUSY) = 140\times10^{-11}tan\beta\times(100GeV/\tilde{m})^2$ with $\tilde{m}$ the mass of the lightest SUSY particle (Chattopadhyay and Nath 1996). The measurement goal is $\Delta a_\mu(\text{expt}) = 40\times10^{-11}$ and should be reached around the year 2001.

3.3 $\mu \rightarrow e\gamma$ decay

The signature of a $\mu \rightarrow e\gamma$ event is a 52.8MeV positron emitted back to back with a 52.8MeV photon. The MEGA experiment at the late Los Alamos Meson Physics Facility (LAMPF) consisted of a magnetic spectrometer to observe the charged final state particle and three pair spectrometers for detecting the photon through its e^+e^- pair creation in lead converters. Random coincidences at high rates are reported as major background.

Data taking is completed and 16% of the data could be analyzed leading to an upper limit on the branching ratio of 3.8×10^{-11} (Cooper*et al.*1997) which slightly improves the value of 4.9×10^{-11} established in a crystal box detector also at LAMPF (Bolton *et al.* 1988).

At PSI new efforts are being discussed to reach a sensitivity of about 5×10^{-14} for this decay mode within the next couple of years. The suggested instruments include solutions like a large solid angle magnetic spectrometer for the e^+ surrounded by a crystal calorimeter for the γ, or liquid Xe calorimeters for the γ and others (Schaaf *et al.* 1998).

It should be noted that the tightest bounds on bileptonic gauge bosons, which are common to many speculative standard model extensions, come from $\mu \rightarrow e\gamma$, if flavour democracy is assumed (Zeller 1998).

3.4 $\mu \rightarrow e$ conversion

Many constraints for speculative models arise from the present experimental bound on the conversion process $\mu + Z \rightarrow e + Z$ (Table 3), which is the tightest for all studied lepton number violatimg (LNV) decays. The variety of theoretically possible processes that can be tested includes, e.g. supersymmetric loop graphs, heavy neutrinos, leptoquarks, compositeness, Higgs bosons and heavy Z' bosons with anomalous couplings. Generally it is more sensitive to new Physics than $\mu \rightarrow e\gamma$ in a wide class of models where the process is generated at the one loop level (Raidaland Santamaria 1997).

The process needs to involve a nucleus to assure elementary conservation laws. If the nucleus is left in its ground state, a conversion event is signaled through the release of a 105MeV electron, which is uniquely distinguishable from normal muon decay electrons ranging up to 53MeV. Among the physically relevant intrinsic background processes is μ decay in the atomic orbit after a muonic atom has been formed, which can release much higher energetic electrons, and radiative muon capture, where the photokinematic end point can be close to the signal electron energy.

The ongoing SINDRUM II experiment uses the world's brightest continuous muon channel π E5 at PSI. Their new results limit the branching ratios $\mu^-\mathrm{Ti} \rightarrow e^+\mathrm{Ca}^{gs}$ to below 1.7×10^{-12} for the Ca nucleus in the ground state (Kaulard *et al.* 1998), $\mu^-\mathrm{Ti} \rightarrow e^+\mathrm{Ca}^{GDR}$ to below 3.6×10^{-11} leaving Ca with giant dipole resonance excitation (Kaulard *et al.* 1998), and $\mu^-\mathrm{Ti} \rightarrow e^-\mathrm{Ti}$ to below 6.1×10^{-13} for Ti in the ground state (Eggli *et al.* 1998). For the ground state processes the nucleons interact coherently which enhances the possible effect. In order to boost accuracy in the near future the SINDRUM II collaboration wants to take advantage in the gain of muon flux through a $\pi - -\mu$ converter, a novel superconducting device in the beam line which collects π's and releases only μ's with very low π contamination. The latter point is essential as π's are a source of potential background due to nuclear reactions. The projections of the collaboration for the achievable limit in the coherent $\mu^-\mathrm{Ti} \rightarrow e^-\mathrm{Ti}$ case are in the 10^{-14} region.

The new Muon Electron Conversion (MECO) experiment proposed at BNL (Molzon *et al.* 1997) (see Figure 17) is very close in its design to a proposal by Lobashev and collaborators for the Moscow Meson Factory. The setup consists of a target station for π/μ production which uses a proton beam from the AGS accelerator, an S-shaped transport and purification section and a detector the basic idea of which is to let electrons from normal muon decay pass without being seen and to observe only the 105MeV signal elec-

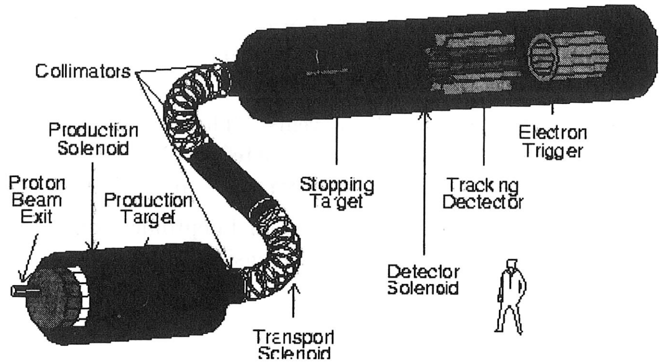

Figure 17. *The MECO experiment planned at BNL (Molzon* et al. *1997).*

trons. The goal is the 10^{-16} level in sensitivity, which will stringently test supersymmetric models; there is an anticipated ultimate capability for 10^{-18} .

3.5 $\mu^+e^- \rightarrow \mu^-e^+$ conversion

The hydrogen-like muonium atom consists of two leptons from different generations. The close confinement of the bound state offers excellent opportunities to explore precisely fundamental electron-muon interactions (Hughes and zu Putlitz 1990). Since the effect of all known fundamental forces in this system are calculable very well mainly in the framework of quantum electrodynamics (QED), it renders the possibility to search sensitively for yet unknown interactions between both particles. A M-$\overline{\text{M}}$-conversion would violate additive lepton family number conservation and is discussed in many speculative theories (see Figure 18). It would be an analogy in the lepton sector to K^0-$\overline{K^0}$ oscillations.

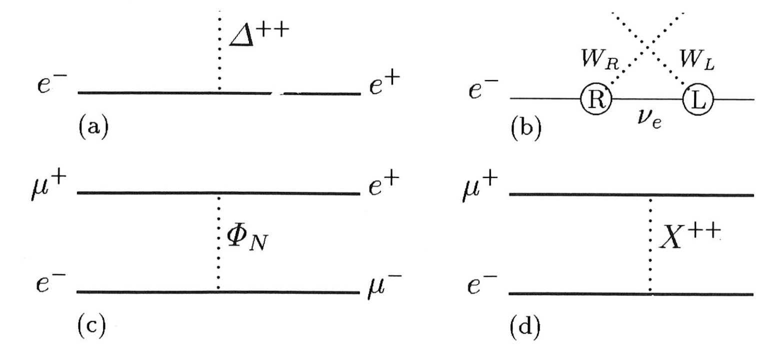

Figure 18. *$M - \overline{M}$ conversion in theories beyond the standard model. The interaction could be mediated by (a) a doubly charged Higgs boson Δ^{++} (Halprin 1982, Herczeg and Mohapatra 1992), (b) heavy Majorana neutrinos (Halprin 1982), (c) a neutral scalar Φ_N (Hou and Wong 1996), e.g. a supersymmetric τ-sneutrino $\tilde{\nu}_\tau$ (Halprin and Masiero 1993, Mohapatra 1992), or (d) a dileptonic gauge boson X^{++} (Fujii* et al. *1994).*

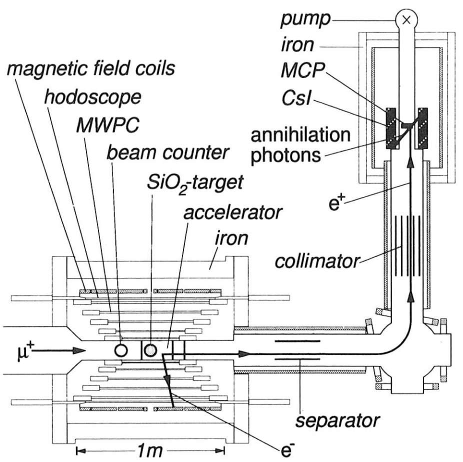

Figure 19. *Top view of the MACS (Muonium Antimuonium Conversion Spectrometer) apparatus at PSI to search for* M – $\overline{\mathrm{M}}$ *conversion (Abela* al. *1996).*

The setup at PSI (Figure 19) (Abela *al.* 1996) is designed to employ the signature developed in a predecessor experiment at LAMPF, which requires the coincident identification of both particles forming the antiatom in its decay (Matthias *et al.* 1991, Willmann and Jungmann 1997). Muonium atoms in vacuum with thermal velocities, which are produced from a SiO_2 powder target, are observed for antimuonium decays. Energetic electrons from the decay of the μ^- in the antiatom can be observed in a magnetic spectrometer at 0.1T, consisting of five concentric multiwire proportional chambers and a 64 fold segmented hodoscope. The positron in the atomic shell of the antiatom is left behind after the decay with 13.5eV average kinetic energy (Chatterjee *et al.* 1992). It can be accelerated to 7keV in a two stage electrostatic device and guided in a magnetic transport system to a position sensitive microchannel plate detector (MCP). Annihilation radiation can be observed in a 12 fold segmented pure CsI calorimeter around it.

The relevant measurements were performed during in total 6 month distributed over 4 years during which 5.7×10^{10} muonium atoms were in the interaction region. One event fell within a 99% confidence interval of all relevant distributions (Figure 20). The expected background due to accidental coincidences is 1.7(2) events. Thus an upper limit on the conversion probability of $\mathrm{P_{M\overline{M}}} \leq 8.2\times10^{-11}/\mathrm{S_B}$ (90% C.L.) was found, where $\mathrm{S_B}$ accounts for the interaction type dependent suppression of the conversion in the magnetic field of the detector due to the removal of degeneracy between corresponding levels in M and $\overline{\mathrm{M}}$. The reduction is strongest for (V±A)×(V±A), where $\mathrm{S_B}$=0.35 (Horrikawa and Sasaki 1996, Wong and Hou 1995). This yields for the traditionally quoted upper limit on the coupling constant in effective four fermion interaction $\mathrm{G_{M\overline{M}}} \leq 3.0\times10^{-3}\mathrm{G_F}(90\% C.L.)$ with $\mathrm{G_F}$ the weak interaction Fermi constant.

This new result, which exceeds bounds from previous experiments (Matthias *et al.* 1991, Gordeev *et al.* 1994) by a factor of 2500 and the one from an early stage of the experiment (Abela *al.* 1996) by 35, has some impact on speculative models. A certain Z_8 model is ruled out with more than 4 generations of particles where masses could be generated radiatively with heavy lepton seeding (Wong and Hou 1994).

A new lower limit of $m_{X^{\pm\pm}} \geq 2.6\mathrm{TeV/c^2} *g_{3l}$ (95% C.L.) on the masses of flavour diagonal bileptonic gauge bosons in GUT models is found well beyond the value derived

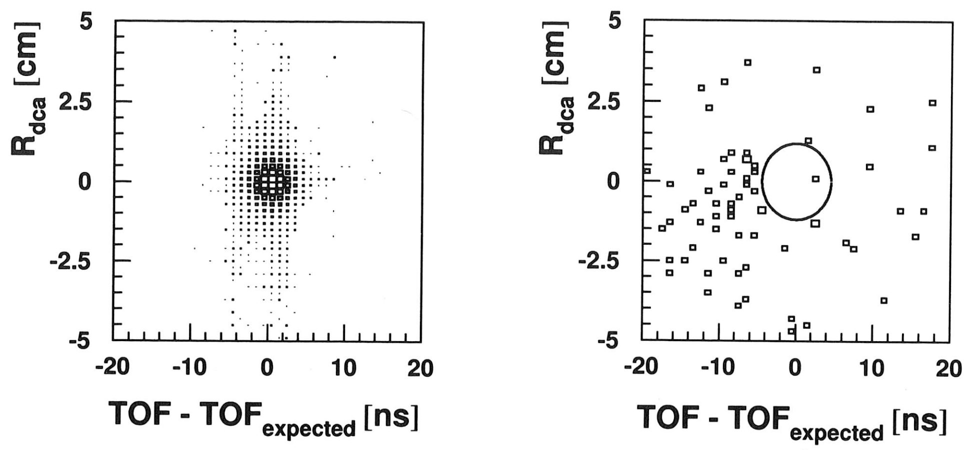

Figure 20. *Time of flight (TOF) and vertex quality for a muonium measurement (left) and the same for all data of the final 4 month search for antimuonium (right). One event falls into the indicated 3 standard deviations area.*

from direct searches, measurements of the muon magnetic anomaly or high energy Bhabha scattering (Fujii H *et al.* 1994, Zeller 1998). Here g_{3l} is of order 1 and depends on the details of the underlying symmetry. For 331 models this translates into $m_{X^{\pm\pm}} \geq 850\mathrm{GeV/c^2}$ which excludes their minimal Higgs version in which an upper bound of $600\mathrm{GeV/c^2}$ has been extracted from an analysis of electroweak parameters (Frampton and Harada 1997 , Frampton 1997). The 331 models may still be viable in some extended form involving a Higgs octet (Frampton 1998). In the framework of R-parity violating supersymmetry (Mohapatra 1992, Halprin and Masiero 1993) the bound on the coupling parameters could be lowered by a factor of 15 to $\mid \lambda_{132}\lambda^*_{231} \mid \leq 3\times10^{-4}$ for assumed superpartner masses of $100\mathrm{GeV/c^2}$. The achieved level of sensitivity allows one to narrow the interval of allowed heavy muon neutrino masses in minimal left-right symmetry (Herczeg and Mohapatra 1992) (where a lower bound on $\mathrm{G_{M\overline{M}}}$ exists, if muon neutrinos are heavier than 35keV) to $\approx 40\mathrm{keV/c^2}$ up to the present experimental bound at $170\mathrm{keV/c^2}$.

In minimal left right symmetric models, in which $\mathrm{M\overline{M}}$ conversion is allowed, the process is intimately connected to the lepton family number violating muon decay $\mu^+ \rightarrow e^+ + \nu_\mu + \overline{\nu}_e$. With the limit achieved in this experiment this decay is not an option for explaining the excess neutrino counts in the LSND neutrino experiment at Los Alamos (Herczeg 1997, Athanassopoulos *et al.* 1994).

The consequences for atomic physics of muonium are such that the expected level splitting in the ground state due to $\mathrm{M} - -\overline{\mathrm{M}}$ interaction is below $1.5\mathrm{Hz}/\sqrt{S_B}$ reassuring the validity of fundamental constants determined in muonium spectroscopy.

A future $\mathrm{M} - -\overline{\mathrm{M}}$ experiment could take particularly advantage of high intense pulsed beams. In contrast to other LNV muon decays, the conversion through its nature as particle-antiparticle oscillation, has a time evolution in which the probability for finding $\overline{\mathrm{M}}$ in the ensemble remaining after muon decay increases quadratically in time, giving the signal an advantage growing in time over major exponentially decaying background (Willmann and Jungmann 1997).

4 Theory of the muonium atom

4.1 One-electron atoms

4.1.1 Dirac theory of the gross and fine structure

The total energy of an electron bound in an one-electron atom (Bethe and Salpeter 1957) can be expressed as

$$\begin{aligned} E_{\text{tot}}(n,j,l,F) &= E_D(n,j) + E_{\text{RM}}(n,j,l) + E_{\text{QED}}(n,j,l) \\ &\quad + E_{\text{HFS}}(n,j,l,F,I) + E_{\text{strong}} + E_{\text{weak}} + E_{\text{exotic}}, \end{aligned} \tag{9}$$

with the principal quantum number n, the electron angular momentum j, the the orbital angular momentum l, the total angular momentum F, and the nuclear Spin I. The dominant part is the Dirac energy $E_D(n,j)$ which is an eigenvalue of the Dirac equation for an electron of rest mass m_e bound to a point-like nucleus of infinite mass and with a charge of $Z\times e$ which creates a potential $V = -Z\alpha/r$,

$$E_D(n,j) = m_e c^2 \left(f(n,j) - 1\right), \tag{10}$$

where m_e represents the electron mass and c is the speed of light. The function $f(n,j)$ is given by

$$f(n,j) = \left[1 + \left(\frac{Z\alpha}{n-\varepsilon}\right)^2\right]^{-1/2} \tag{11}$$

with

$$\varepsilon = j + \frac{1}{2} - \sqrt{\left(j+\frac{1}{2}\right)^2 - (Z\alpha)^2}. \tag{12}$$

The Dirac theory describes the system to the fine structure level. The finite nuclear mass is taken into account by a reduced mass E_{RM} correction to E_D. This contribution is composed of two parts

$$E_{\text{RM}}(n,j,l) = E_{\text{NRRM}} + E_{\text{RRM}}(n,j,l). \tag{13}$$

The first one describes the classical nonrelativistic reduced mass term

$$E_{\text{NRRM}} = \left(\frac{m_r}{m_e} - 1\right) E_D \tag{14}$$

and the second one is the relativistic reduced mass effect which arises in the full relativistic treatment of the two-body problem

$$E_{\text{RRM}}(n,j,l) = -\frac{m_r^2 c^2}{2(m_e + m_N)} \left(f(n,j) - 1\right)^2 + \frac{(Z\alpha)^4 m_r^3 c^2}{2n^3 m_N^2} \left(\frac{1}{j+\frac{1}{2}} - \frac{1}{l+\frac{1}{2}}\right)(1-\delta_{l0}), \tag{15}$$

and which depends also on the orbital angular momentum l. Higher order terms are neglected. The reduced mass is defined by the expression

$$m_r = m_e m_N/(m_e + m_N). \tag{16}$$

Additional corrections will be discussed below. They arise from quantum electrodynamical effects ($E_{\text{QED}}(n,j,l)$), the hyperfine interaction ($E_{\text{HFS}}(n,j,l,F)$), the strong interaction (E_{strong}, which enters indirectly through the QED effect of vacuum polarisation), the weak interaction (E_{weak}, due to Z boson exchange), and possibly from exotic processes (E_{exotic}), which are so far not provided in standard theory.

4.1.2 QED corrections – Lamb shift

The deviations of the real atomic levels from the fine structure level energies as predicted in the Dirac theory and corrected for a finite nuclear mass can be calculated to high precision within the framework of QED. They are referred to as the Lamb shift and originate mainly from radiative corrections to the electron propagator, from vacuum polarisation, from nuclear motion and from the effects of nuclear motion in radiative corrections (combined effects). Nuclear motion can be taken into account in a first approximation using the reduced mass concept. Further corrections (in higher than first order of the mass ratio) due to the dynamics of the nuclear motion in the system are traditionally called recoil corrections. The QED corrections depend most of all on the probability density of the electron wave function at the origin (important for the for s-states) and the expectation value for r^{-3}, both of which are proportional to $1/n^3$. The theoretical work over the last four decades has been reviewed recently by Sapirstein and Yennie (1990) and Yennie (1992). Therefore we quote the major results.

Figure 21. *Lowest order contributions to the Lamb shift. (a) Electron self energy. (b) Vacuum polarisation correction to the potential. The heavy lines represent the electron in an external static nuclear field.*

For electronic atoms the dominating contribution arises from one-photon radiative corrections to the electron propagator (self energy and anomalous magnetic moment) and from vacuum polarisation effects to the potential. (In the case of muonic atoms the situation is reversed: The vacuum polarisation dominates over the self energy part.This makes muonic atoms particularly interesting for studying vacuum polarisation effects.) In the non-recoil limit (derived for infinite nuclear mass, and with the additional inclusion of the reduced mass dependence, which for example describe the scaling of the wave functions, and mass corrections for the magnetic moments) they amount to

$$\Delta E_{\text{one-loop}}(n,j,l) = \frac{m_e c^2 \alpha (Z\alpha)^4}{\pi n^3} F_n(Z\alpha) \qquad (17)$$

where

$$\begin{aligned} F_n(Z\alpha) &= A_{40}(n) + A_{41}(n)\ln(\sigma(Z\alpha)^{-2}) + (Z\alpha)A_{50}(n) \\ &\quad + (Z\alpha)^2\left[G(n,Z\alpha) + A_{61}(n)\ln(\sigma(Z\alpha)^{-2}) + A_{62}(n)\ln^2(\sigma(Z\alpha)^{-2})\right] \qquad (18) \end{aligned}$$

with $\sigma = m_e/m_r$. The coefficients A_{pq} from expansions in $\alpha^r(Z\alpha)^p \ln^q(Z\alpha)^{-2}$, where r corresponds to the number of emitted and re-absorbed virtual photons, have been calculated analytically:

$$A_{40}(nS) = \left[\frac{10}{9}\left\{-\frac{4}{15}\right\}_{\text{VP}} - \frac{4}{3}\ln(k_0(nS))\right]\left(\frac{m_r}{m_e}\right)^3$$

$$
\begin{aligned}
A_{40}(n, l \neq 0) &= \left[\frac{C_{lj}}{2(2l+1)}\right]\left(\frac{m_r}{m_e}\right)^2 + \left[-\frac{4}{3}\ln k_0(n, l \neq 0)\right]\left(\frac{m_r}{m_e}\right)^3 \\
A_{41}(nS) &= \left[\frac{4}{3}\right]\left(\frac{m_r}{m_e}\right)^3 \\
A_{50}(nS) &= 4\pi\left[\frac{139}{128} + \left\{\frac{5}{192}\right\}_{VP} - \frac{1}{2}\ln 2\right]\left(\frac{m_r}{m_e}\right)^3 \\
A_{61}(1S) &= \left[\frac{28}{3}\ln 2 - \frac{21}{20} - \frac{2}{15}\right]\left(\frac{m_r}{m_e}\right)^3 \\
A_{61}(2S) &= \left[\frac{16}{3}\ln 2 + \frac{67}{30}\left\{-\frac{2}{15}\right\}_{\mathrm{VP}}\right]\left(\frac{m_r}{m_e}\right)^3 \\
A_{61}(2P_{1/2}) &= \left[\frac{103}{180}\right]\left(\frac{m_r}{m_e}\right)^3 \\
A_{61}(2P_{3/2}) &= \left[\frac{29}{90}\right]\left(\frac{m_r}{m_e}\right)^3 \\
A_{62}(nS) &= -[1]\left(\frac{m_r}{m_e}\right)^3,
\end{aligned}
\tag{19}
$$

where $\ln k_0(n, l)$ denotes the Bethe logarithm with the values (Huff 1969)

$$
\begin{aligned}
\ln k_0(1S) &= 2.984\,128\,555\,9(3) \\
\ln k_0(2S) &= 2.811\,769\,893\,2(5) \\
\ln k_0(2P) &= -0.030\,016\,708\,9(3).
\end{aligned}
\tag{20}
$$

The expressions arising from vacuum polarisation are indicated in $\{\}_{\mathrm{VP}}$ brackets. The non-logarithmic terms $G(n, Z\alpha)$ of order $m_e\alpha(Z\alpha)^6$ and higher orders are available numerically:

$$
G(n, Z\alpha) = [G_{\mathrm{SE}}(n, Z\alpha) + G_{\mathrm{VP}}(n, Z\alpha)]\left(\frac{m_r}{m_e}\right)^3 \tag{21}
$$

with the self energy contributions

$$
\begin{aligned}
G_{\mathrm{SE}}(1S) &= [-30.3 \pm 1.5] \\
G_{\mathrm{SE}}(2S) &= [-32.1 \pm 1.5] \\
G_{\mathrm{SE}}(2P_{1/2}) &= [-0.8 \pm 0.3] \\
G_{\mathrm{SE}}(2P_{3/2}) &= [-0.5 \pm 0.3]
\end{aligned}
\tag{22}
$$

(23)

and the vacuum polarisation terms

$$
\begin{aligned}
G_{\mathrm{VP}}(1S) &= \left[\frac{4}{15}\ln 2 - \frac{1289}{1575} + \left(\frac{19}{45} - \frac{\pi^2}{27}\right) + O(Z\alpha)\right] \\
G_{\mathrm{VP}}(2S) &= \left[-\frac{743}{900} + \left(\frac{19}{45} - \frac{\pi^2}{27}\right) + O(Z\alpha)\right] \\
G_{\mathrm{VP}}(2P_{1/2}) &= \left[-\frac{9}{140} + O(Z\alpha)\right] \\
G_{\mathrm{VP}}(2P_{3/2}) &= \left[-\frac{1}{70} + O(Z\alpha)\right],
\end{aligned}
\tag{24}
$$

where $C_{lj} = 2(j - l)/(j + 1/2)$. Recently analytical expressions for some of the lowest order contributions to $G(n, Z\alpha)$ for the 1S state were presented by Pachuki (Pachuki 1992, Pachucki 1998). He obtains $G(1S) = -31.04309(1)$ in agreement with the results in Equation (24).

The Lamb shift is largest for s-states, since they have a finite probability density at the origin. Not included in the expressions of Equation (19,24) are vacuum polarisation loops formed by particles different from the electron (muons, taus, hadrons). In general they contribute proportional to the inverse square of their masses. For example muons contribute $(m_e/m_\mu)^2 \approx 2.3\times10^{-5}$ as much as electrons. For the classical Lamb shift this is of the order of 600Hz. The hadronic contribution to the s-states is of relative order $\alpha(Z\alpha)^4(m_e/m_\pi)^2$ (Brodsky and Drell 1970), where m_π is the pion mass. Fortunately it can be calculated in detail without knowledge of the underlying strong interaction theory using input data from the measured ratio of the total cross sections $\sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$.

So far the expressions are valid for small m_e/m_N ratios only. In the case of positronium one needs to take into account the self energy of the positron as well as the spin orbit coupling effects caused by the anomalous magnetic moment of the positron (Sapirstein and Yennie 1990). The positronium atom further needs corrections for the hyperfine splitting of order $\alpha^2 R_\infty/n^3$. For the triplet states the annihilation into a single virtual photon causes an upward shift of the order $\alpha^2 R_\infty/2n^3$. The relativistic recoil (also known as Salpeter correction) contributes to relative order $(Z\alpha)^5 m_r^3/(m_e m_N)$ and can be derived from the Bethe-Salpeter equation (Salpeter 1952). The Breit equation (Breit and Meyerott 1947) used formerly is not appropriate since it misses terms of that order. We have

$$\begin{aligned}\Delta E_{\text{rel-rec}}(n,l) \;=\; & \frac{m_r^3c^2}{m_e m_N}\frac{(Z\alpha)^5}{\pi n^3}\left\{\frac{2}{3}\delta_{l0}\ln\left(\frac{1}{Z\alpha}\right) - \frac{8}{3}\ln k_0(n,l) - \frac{1}{9}\delta_{l0} - \frac{7}{3}a_n\right.\\ & \left. - \frac{2}{m_N^2 - m_e^2}\delta_{l0}\left[m_N^2\ln\frac{m_e}{m_r} - m_e^2\ln\frac{m_N}{m_r}\right]\right\}\\ & \times\frac{m_e^2c^2}{m_N}\frac{(Z\alpha)^6}{n^3}\left(4\ln 2 - \frac{7}{2}\right),\end{aligned}$$

where the state dependent function a_n (Erickson 1977) is

$$a_n = -2\left[\ln\frac{2}{n} + \left(1 + \frac{1}{2} + ... + \frac{1}{n}\right) + 1 - \frac{1}{2n}\right]\delta_{l0} + \frac{1 - \delta_{l0}}{l(l+1)(2l+1)}. \qquad (25)$$

Recoil terms of order $(Z\alpha)^6 m_e^2/m_N$ have been recently presented by Erickson and Grotch (1988) and Doncheski *et al.* (1991).

The precision achieved both in experiment and theory has reached a level where the separate treatment of radiative and recoil corrections does not provide results of sufficient accuracy. Combined effects have to be taken into account. The effect of nuclear recoil in radiative corrections (radiative-recoil) has been calculated by Bhatt and Grotch (1985) to be

$$\Delta E_{\text{rad-rec}}(n,l) = \frac{\alpha(Z\alpha)^5 m_e^2 c^2}{n^3 m_N}\left(\frac{35}{4}\ln 2 - \frac{7333}{960} - 0.415 \pm 0.004\right)\delta_{l0}. \qquad (26)$$

The inverse m_N dependence of the relativistic-recoil and the radiative recoil terms causes a significant difference in the Lamb shift for hydrogen and muonium where they are larger by the ratio $m_p/m_\mu \approx 8.9$ of the proton to the muon mass.

Higher order corrections of order α^2 can be summarised in a way analogous to the one-loop terms as

$$\Delta E_{\text{two-loop}}(n,l,j) = \frac{m_e \alpha^2 (Z\alpha)^4}{\pi^2 n^3} H_n(Z\alpha). \tag{27}$$

The functions $H_n(Z\alpha)$ are:

$$\begin{aligned} H_{\text{s-states}}(Z\alpha) &= \left[-\frac{4358}{1296} - \frac{10}{27}\pi^2 + \frac{3}{2}\pi^2 \ln 2 - \frac{9}{4}\zeta(3)\right] \left(\frac{m_r}{m_e}\right)^3 \\ H_{\text{non s-states}}(Z\alpha) &= \left[+\frac{197}{72} + \frac{1}{6}\pi^2 - \pi^2 \ln 2 + \frac{3}{2}\zeta(3)\right] \frac{C_{lj}}{2(2l+1)} \left(\frac{m_r}{m_e}\right)^2 \end{aligned} \tag{28}$$

with the Riemann ζ-function $\zeta(x) = \sum_{\nu=1}^{\infty} 1/\nu^x$.

Two-loop binding corrections of order $\alpha^2(Z\alpha)^5$ contribute –296.90(4)kHz for the 1s and –37.112(5)kHz for the 2s state (Eides *et al.* 1992, Eides 1996, Pachuck 1994). Some additional higher order radiative corrections have been recently calculated (Karshenboim 1993), they contribute at the few kHz and below level to the 1s and 2s Lamb shifts and are of no concern for muonium yet.

The finite charge distribution within the nuclei of hydrogen-like systems with hadronic nuclei give rise to a shift mainly for the s-states since they have the largest probability density within the nuclei. Assuming a spherical charge distribution of radius r_N it amounts for nonrelativistic systems to

$$\Delta E_{\text{finite-size}}(n) = \frac{2}{3n^3}(Z\alpha)^4 \left(\frac{m_r}{m_e}\right)^3 \frac{\langle r_N^2 \rangle}{\lambda_c^{\ 2}}. \tag{29}$$

In principle, an influence of deviations from the spherical nuclear shape must be expected. For most practical cases, however, the mean square charge radius can be chosen for $< r_N^2 >$. Combined effects of finite nuclear size with vacuum polarisation and radiative corrections are of the order of $\alpha(Z\alpha)^5 \times (m_r/m_e)^3 \times (< r_N^2 > / \lambda_c^2)$ and can be neglected. (For muonic atoms the nuclear-size effects are more important due to the larger overlap of the muon wave functions with the nuclear charge density compared to corresponding electronic states.)

4.1.3 Hyperfine structure

The interaction of the magnetic moment μ_N of the nucleus with spin I with the electrons magnetic moment μ_e gives rise to the hyperfine structure splitting which is to a good approximation given by the Fermi formula (Fermi 1930):

$$E_{\text{HFS}}(n,j,l,F,I) = (Z\alpha)^2 Z g_I \frac{m_e}{m_N}(1+\varepsilon_{QED}) \left(\frac{m_r}{m_e}\right)^3 \frac{hcR_\infty}{n^3} \frac{F(F+1)-I(I+1)-j(j+1)}{j(j+1)(2l+1)}. \tag{30}$$

Here the term ε_{QED} takes into account relativistic effects, radiative and recoil corrections as well as the finite nuclear size and nuclear polarisability:

$$\varepsilon_{\text{QED}} = \varepsilon_{\text{rad}} + \varepsilon_{\text{rec}} + \varepsilon_{\text{rad-rec}} + \varepsilon_{\text{nucl-size}} + \varepsilon_{\text{nucl-pol}} \tag{31}$$

constant		value	reference
Rydberg constant R_∞	[cm^{-1}]	10 973 731.568 639(91)	(de Beauvoir *et al.* 1997 Udem *et al.* 1997)
fine structure α		1 / 137.0360037(33)	(Jeffrey *et al.* 1997 Cage *et al.* 1989
electron mass m_e	[MeV/c^2]	0.51099906(15)	(Cohen and Taylor 1987)
muon mass m_μ	[MeV/c^2]	105.658389(34)	(Cohen and Taylor 1987)
proton mass m_p	[MeV/c^2]	938.27231(28)	(Cohen and Taylor 1987)
deuteron mass m_d	[MeV/c^2]	1875.61339(57)	(Cohen and Taylor 1987)
electron g-factor g_e		2.002319304386(20)	(Cohen and Taylor 1987)
muon g-factor g_μ		2.002331846(17)	(Cohen and Taylor 1987)
proton g-factor g_p		5.585694772(126)	(Cohen and Taylor 1987)
deuteron g-factor g_d		1.714876460(48)	(Cohen and Taylor 1987)
mass ratio m_μ/m_e		206.768262(30)	(Cohen and Taylor 1987)
mass ratio m_p/m_e		1836.152701(37)	(Cohen and Taylor 1987)
mass ratio m_d/m_e		3670.483014(75)	(Cohen and Taylor 1987)
proton size† r_p	[fm]	0.862(12)	(Simon *et al.* 1980)
deuteron size r_d	[fm]	2.116(12)	(Ericson *et al.* 1984)
speed of light c	[m/s]	299 792 458	(Cohen and Taylor 1987)
Planck constant h	[eVs]	4.1356692(12)$\times 10^{-15}$	(Cohen and Taylor 1987)
Fermi const. $G_F/(\hbar c)^3$	[GeV^{-2}]	1.16639(1)$\times 10^{-5}$	(Caso *et al.* 1998)
$\sin^2 \Theta_W(\overline{MS})$		0.23124(24)	(Caso *et al.* 1998)
muon lifetime τ_μ	[μs]	2.19703(4)	(Caso *et al.* 1998)

Table 4. *Fundamental constants necessary for the calculations of bound two body level structures and transition frequencies. [†For the mean square charge radius of the proton the results of the most recent electron scattering experiment were taken. An earlier published value (Hand* et al. *1963) has been shown to be wrong due to an error in the extrapolation of experimental data to low momentum transfer. A re-analysis yields agreement with the new results (Rosenfelder 1992).]*

For the muonium atom, where nuclear structure effects are absent, the ground state splitting between the F=0 and F=1 levels can be expressed as

$$\Delta\nu^M_{\rm HFS} = \frac{16}{3}(Z\alpha)^2 R_\infty \frac{\mu_\mu}{\mu_B}\left[1+\frac{m_e}{m_\mu}\right]^{-3}(1+\varepsilon_{\rm rad}+\varepsilon_{\rm rec}+\varepsilon_{\rm rad-rec}) + \Delta\nu_{\rm strong}+\Delta\nu_{\rm weak}+\Delta\nu_{\rm exotic}. \quad (32)$$

Using

$$R_\infty = \frac{\alpha^2 m_e c^2}{2h} \quad (33)$$

we have

$$\Delta\nu^M_{\rm HFS} = \frac{8}{3}(Z\alpha)^4 c^2 (\frac{m_e}{h})\frac{\mu_\mu}{\mu_B}\left[1+\frac{m_e}{m_\mu}\right]^{-3}(1+\varepsilon_{\rm rad}+\varepsilon_{\rm rec}+\varepsilon_{\rm rad-rec}) + \Delta\nu_{\rm strong}+\Delta\nu_{\rm weak}+\Delta\nu_{\rm exotic}, \quad (34)$$

which is a powerful relation for an accurate determination of the fine structure constant, as the quantity m_e/h can be determined accurately from measurements of the neutron de Broglie wavelength and well known mass ratios.

The radiative corrections to the hyperfine splitting of one-electron atoms are in low orders

$$\begin{aligned}\varepsilon_{\text{rad}} &= (1+a_\mu)\left\{1+\frac{3}{2}(Z\alpha)^2 + a_e + \alpha(Z\alpha)\left(\ln 2 - \frac{5}{2}\right)\right.\\ &\quad - \frac{8\alpha(Z\alpha)^2}{3\pi}\ln(Z\alpha)\left(\ln(Z\alpha/4)+\frac{281}{480}\right)\\ &\quad \left.+\frac{\alpha(Z\alpha)^2}{\pi}(16.9042\pm 0.11)+\frac{\alpha^2(Z\alpha)}{\pi}(0.7717\pm 4)\right\}\ .\end{aligned}$$

The nuclear recoil contributes

$$\varepsilon_{\text{rec}} = -\frac{3Z\alpha}{\pi}\frac{m_e m_\mu}{m_\mu^2 - m_e^2}\ln\frac{m_\mu}{m_e} + \frac{(m_r Z\alpha)^2}{m_e m_\mu}\left[2\ln\frac{1}{2Z\alpha} - 6\ln 2 + \frac{65}{18}\right]. \tag{35}$$

The radiative recoil correction to the hyperfine interval is

$$\begin{aligned}\varepsilon_{\text{rad-rec}} &= \frac{\alpha(Z\alpha)m_e}{\pi^2 m_\mu}\left\{-2\ln^2\frac{m_\mu}{m_e}+\frac{13}{12}\ln\frac{m_\mu}{m_e}\frac{21}{2}+\zeta(3)+\zeta(2)\right.\\ &\quad \left.+\frac{35}{9}+\frac{\alpha}{\pi}\left[-\frac{4}{3}\ln^3\frac{m_\mu}{m_e}+\frac{4}{3}\ln^2\frac{m_\mu}{m_e}+O(\ln\frac{m_\mu}{m_e})\right\}\right).\end{aligned} \tag{36}$$

From the hadronic vacuum polarisation there is a shift due to strong interaction.

$$\begin{aligned}\Delta\nu_{\text{strong}} &= \frac{8}{3}(Z\alpha)^4 c^2(\frac{m_e}{h})\frac{\mu_\mu}{\mu_B}\left[1+\frac{m_e}{m_\mu}\right]^{-3}\\ &\quad \times 1.91(0.26)\times\frac{\alpha(Z\alpha)m_e}{\pi^2 m_\mu}\left(-2\ln^2\frac{m_\mu}{m_e}+\frac{13}{12}\ln\frac{m_\mu}{m_e}\frac{21}{2}+\zeta(3)+\zeta(2)\right),\end{aligned} \tag{37}$$

which amounts to 250Hz.

4.2 Weak interaction

The weak interaction through the exchange of a Z boson gives rise to an additional contribution to the level energies. Because of the very short range of the weak interaction due to the large mass ($\approx 91GeV/c^2$) of the Z boson the interaction can be modeled as a point current-current interaction with an effective vector-axial vector type Hamiltonian (Bég and Feinberg 1974, Starshenko and Faustov 1983, Hinds 1988, Eides 1996):

$$\begin{aligned}H_{\text{eff}} &= \tfrac{G_F}{\sqrt{2}}\textstyle\sum_N \left(C_{eN}^{VV}\,\overline{\Psi}_e\gamma^\rho\Psi_e\,\overline{\Psi}_N\gamma_\rho\Psi_N + C_{eN}^{VA}\,\overline{\Psi}_e\gamma^\rho\Psi_e\,\overline{\Psi}_N\gamma_\rho\gamma_5\Psi_N\right.\\ &\quad \left.+C_{eN}^{AV}\,\overline{\Psi}_e\gamma^\rho\gamma_5\Psi_e\,\overline{\Psi}_N\gamma_\rho\Psi_N + C_{eN}^{AA}\,\overline{\Psi}_e\gamma^\rho\gamma_5\Psi_e\,\overline{\Psi}_N\gamma_\rho\gamma_5\Psi_N\right).\end{aligned} \tag{38}$$

Here G_F is the Fermi coupling constant of the weak interaction, Ψ_e describes the electron wave function and Ψ_N runs over all the nucleons in the system. Although the fundamental weak coupling of the electron is to the quarks within the nucleus (in case of hadronic nuclei), at low energies the nucleons may be considered as fundamental (Hinds 1988). In the case of muonium Ψ_N represents the muon. The coupling constants for different nuclei are listed in Table 5. They are valid in the limit of low momentum transfer and they can be expressed in terms of the electroweak mixing angle (Weinberg angle) Θ_W and the ratio of the axial vector to vector amplitude g_A/g_V with the experimental value $g_A/g_V = 1.2573(28)$ (Caso 1998).

N	C_{eN}^{VV}	C_{eN}^{VA}	C_{eN}^{AV}	C_{eN}^{AA}
p	$-\frac{(1-4\sin^2\Theta_W)^2}{2}$	$\frac{g_A}{g_V}\frac{(1-4\sin^2\Theta_W)}{2}$	$\frac{(1-4\sin^2\Theta_W)}{2}$	$-\frac{g_A}{2g_V}$
n	$\frac{(1-4\sin^2\Theta_W)}{2}$	$-\frac{g_A}{g_V}\frac{(1-4\sin^2\Theta_W)^2}{2}$	$-\frac{1}{2}$	$\frac{g_A}{2g_V}$
μ	$-\frac{(1-4\sin^2\Theta_W)^2}{2}$	$\frac{(1-4\sin^2\Theta_W)}{2}$	$\frac{(1-4\sin^2\Theta_W)}{2}$	$-\frac{1}{2}$

Table 5. *Coupling constants for Z boson exchange between electron e and "nucleus" N in hydrogen-like systems in the limit of low momentum transfer (Hinds 1988).* Θ_W *is the Weinberg angle (weak mixing angle).* g_A/g_V *is the ratio of the axial vector to vector coupling amplitude from neutron* β*-decay.*

The Parity violating vector–axialvector (VA) and axialvector–vector (AV) terms will not shift atomic energy levels to first order perturbation theory. Only the parity conserving weak interactions can contribute significantly to the level energies. Theoretical work has been performed for the s-states and for the hyperfine structure intervals of hydrogen and hydrogen-like atoms by Bég and Feinberg (1974). Independent calculations by Starshenko and Faustov (1983) on the weak effects on the hyperfine structure yield agreeing results.

4.2.1 Weak effects on s-States

The vector-vector (VV) coupling causes a common shift of the s-state sublevels, since the vectors current $\overline{\Psi}_N\gamma_\rho\Psi_N$ is independent of nuclear spin. For the nS states in hydrogen-like atoms with natural nuclei there is a significant shift (Bég and Feinberg 1974)

$$\Delta E_{\text{weak}}^{Z,A}(nS) = \left(\frac{Z^3}{n^3}\right)\frac{\sqrt{2}G_F m_e^2\alpha^2 m_e c^2}{\pi^2}\left[Z(2C_{ep}^{VV}+C_{en}^{VV})+(A-Z)(C_{ep}^{VV}+2C_{en}^{VV})\right]. \tag{39}$$

The expression reflects the basic coupling to the up- and down-quarks in the protons and neutrons. For atomic hydrogen the weak interaction shift amounts to

$$\Delta E_{\text{weak}}^{H}(nS) = \frac{7.2}{n^3}(2C_{ep}^{VV}+C_{en}^{VV})\ \text{kHz}, \tag{40}$$

and for deuterium to

$$\Delta E_{\text{weak}}^{D}(nS) = \frac{21.6}{n^3}(C_{ep}^{VV}+C_{en}^{VV})\ \text{kHz}. \tag{41}$$

In the muonium atom the coupling is between one electron and one muon. One finds

$$\Delta E^{M}_{\text{weak}}(nS) = -\left(\frac{1}{n^3}\right)\frac{\sqrt{2}G_F m_e^2\alpha^2 m_e c^2}{\pi^2}C^{VV}_{e\mu} = -\frac{7.2}{n^3}(C^{VV}_{e\mu})\ \text{kHz}. \tag{42}$$

The contributions to the 1S-2S energy separations are 190Hz, 613Hz and 24Hz for hydrogen, deuterium and muonium respectively. The accurate most recent 1S-2S measurements in atomic hydrogen (Udem *et al.*) combined with the uncertainty of the QED theory establish an upper bound of $|2C^{VV}_{ep} + C^{VV}_{en}| \leq 16$.

4.2.2 Weak Effects on hyperfine structure

The axialvector–axialvector (AA) term contributes to the ground state hyperfine structure splitting, since the axial current $\overline{\Psi}_N\gamma_\rho\gamma_5\Psi_N$ is proportional to the nuclear spin. The hyperfine splitting (Bég and Feinberg 1974, Starshenko and Faustov) for hydrogen and for muonium are affected significantly different by the weak interaction due to the different muon–electron and nucleus–electron couplings. For the shift $\delta\nu^{H}_{\text{HFS,weak}}$ in hydrogen there there is a relation

$$\begin{aligned}\frac{\delta\nu^{H}_{\text{HFS,weak}}}{\Delta\nu^{H}_{\text{HFS}}} &= 12\frac{G_F M_N^2}{16\sqrt{2}\pi^2\alpha^2}\frac{m_e}{m_p}\left[C^{AA}_{ep}(1+\frac{g_A}{g_V}) + C^{AA}_{en}(1-\frac{g_A}{g_V})\right]/(1+\kappa_p) \\ &\approx 2.2\times10^{-6}(C^{AA}_{ep} - \frac{1}{9}C^{AA}_{en}),\end{aligned} \tag{43}$$

where M_N is the nucleon mass and κ_p=1.79. Since the experimental error is far below the theoretical uncertainty of $\approx$ 1ppm one finds $|C^{AA}_{ep} - \frac{1}{9}C^{AA}_{en}| \leq 0.5$. Unfortunately, no improvement can be expected in the future, unless there will be a better understanding of the proton's internal structure from both the experimental and the theoretical side.

In the case of muonium one finds for the parity conserving weak effect

$$\delta\nu^{M}_{\text{HFS,weak}} = \frac{2\sqrt{2}G_F m_e^2\alpha^2 m_e c^2}{\pi^2}\times C^{AA}_{e\mu}, \tag{44}$$

which yields with $C^{AA}_{e\mu}$ from Table 5

$$\delta\nu^{M}_{\text{HFS,weak}} = 130\times C^{AA}_{e\mu}\ \text{Hz} \ = \ -65\ \text{Hz}. \tag{45}$$

The present uncertainty arising from the QED calculations for $\Delta\nu^{M}_{\text{HFS}}$ and the newest experimental result (Section 5.1.1) fixes an upper limit at $|C^{AA}_{e\mu}| \leq 4$. Significant improvements in theory are possible with an improved muon mass and would reduce this limit further.

Note that the signs of the weak effects in hydrogen and muonium are opposite. This was discovered only recently by Eides (1996): If both particles in the atomic system are different components of the same SU(2) doublet, as it is the case for hydrogen, the signs of the electromagnetic and the weak interaction are the same. Since for the upper and lower components of weak doublets the weak neutral current has opposite sign and the neutral current interaction is identical for particles and antiparticles, the levels of muonium, which consists of the antiparticle μ^+ and e^-, are shifted with an opposite sign compared to hydrogen.

4.3 Exotic Interactions

Exotic interactions between electron and "nucleus" can contribute to the level energies. Therefore precise measurements can provide stringent bounds on the strength of such processes. Of particular interest in this connection is for the muonium atom the possibility of a spontaneous muonium to antimuonium conversion.

Various possibilities have been discussed in theory which could explain muonium-antimuonium conversion. All proposed mechanisms of interest can be described by an effective V-A Hamiltonian (Halprin 1982, Herczeg and Mohapatra 1992)

$$H_{M\overline{M}} = \frac{G_{M\overline{M}}}{\sqrt{2}} \overline{\Psi}_\mu \gamma^\lambda (1+\gamma^5) \Psi_e \overline{\Psi}_\mu \gamma_\lambda (1+\gamma^5) \Psi_e + \mathrm{hc}, \qquad (46)$$

where $G_{M\overline{M}}$ is the coupling constant. In a field free environment the probability of a system which starts as pure muonium to decay as antimuonium is

$$P_{M\overline{M}} = 2.6\times 10^{-5} \left(\frac{G_{M\overline{M}}}{G_F} \right)^2 . \qquad (47)$$

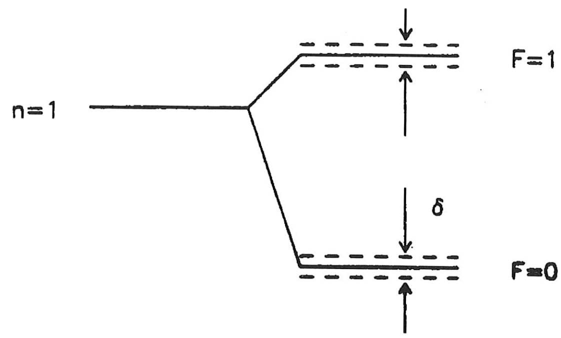

Figure 22. *Possible splitting δ of muonium hyperfine levels due to muonium-antimuonium conversion.*

The ground state of the coupled muonium-antimuonium system has eight energy eigenstates which are different from the four eigenstates of the uncoupled muonium and antimuonium atoms. Their energies have been calculated by Schäfer (Schäfer 1988, Matthias 1991), as well as Wong and Hou (1995) and Horrikawa and Sasaki (1996). In the absence of external fields the muonium-antimuonium oscillations cause a splitting δ of the hyperfine levels of s-states of

$$\delta_{M\overline{M}}(nS) = \langle M | H_{M\overline{M}} | \overline{M} \rangle = \frac{519}{n^3} \times (G_{M\overline{M}}/G_F) \ \mathrm{Hz}. \qquad (48)$$

The effect is shown qualitatively in Figure 22. For the present experimental upper limit on $G_{M\overline{M}}$ of $3\times 10^{-3} G_F$(90%C.L.) (Meyer *et al.* 1997), this amounts to $\delta_{M\overline{M}}(1S) \leq$ 1.5Hz in the ground state.

4.4 Theoretical values for muonium transition frequencies

In muonium the 1s-2s level separation can be calculated using the fundamental constants of Table 4 and is

$$\Delta\nu_{1s-2s}(\text{theory}) = 2\,455\,528\,934.1(1.4)\text{MHz}(0.6\text{ppb}). \tag{49}$$

For the hyperfine splitting the latest compilation of all theoretical results (Karshenboim *et al.* 1992, Eides 1996, Karhenboim 1996a, Pachucki 1996, Kinoshita and Nio 1997 , Blundell *et al.* 1997) yields

$$\Delta\nu_{\text{HFS}}(\text{theory}) = 4\,463\,302\,649(517)(34)(<100)\text{Hz (120ppb)} \tag{50}$$

(Kinoshita 1998). The first uncertainty arises from the muon magnetic moment, the second one from the fine structure constant (from quantum Hall effect) and the last one is due to uncalculated higher order terms. For systems with hadronic nuclei the structure of these particles has to be taken into account. The influence on the hyperfine structure splitting consists of two components: the effects of the charge distribution and effects of the dynamics of the charge carrying constituents within the proton. In hydrogen the size effects together with additional recoil contributions amount to $\approx$ 33(1)ppm (Yennie 1992). The nuclear polarisation effects are not well known. Their contribution is estimated to be less than 4ppm (Hughes and Kuti 1983). Since muonium is free of these nuclear complications, the hyperfine structure splitting of the atom is ideally suited for testing bound state QED, for investigating contributions from the weak interaction and for searching possible exotic interactions. With confidence in the theoretical calculations one can extract accurate values for the fundamental constants that appear in the equations.

5 Microwave spectroscopy of muonium

5.1 Ground state hyperfine structure splitting

5.1.1 Ground State in external magnetic fields

The perturbation Hamiltonian describing the n =1 ground state in an external magnetic field **B** is (Hughes and Kinoshita 1977)

$$H_Z = A_{\text{HFS}}\mathbf{I}_\mu \cdot \mathbf{J} + \mu_B g_J \mathbf{J} \cdot \mathbf{B} - \frac{m_e}{m_\mu}\mu_B g'_\mu \mathbf{I}_\mu \cdot \mathbf{B}, \tag{51}$$

where A_{HFS} is the ground state hyperfine structure splitting $h\Delta\nu^M_{\text{HFS}}$, $\mathbf{I}_\mu$ is the muon spin operator, **J** is the electron total angular momentum, μ_B is the Bohr magnetron, and g_J and g'_μ are the gyromagnetic ratios of the electron and the muon in the muonium atom which differ from the free particle values g_e and g_μ due to relativistic binding corrections (Grotch and Hegstrom 1971)

$$g'_\mu = g_\mu\left[1 - \frac{\alpha^2}{3} + \frac{\alpha^2}{2}\frac{m_e}{m_\mu}\right], \quad g_J = g_e\left[1 - \frac{\alpha^2}{3} + \frac{\alpha^2}{2}\frac{m_e}{m_\mu} + \frac{\alpha^3}{4\pi}\right]. \tag{52}$$

The behaviour of the magnetic sublevels of the F=1 and F=0 hyperfine states in an external magnetic field **B** is expressed by the Breit-Rabi formula (Hughes and Kinoshita 1977)

$$E(n{=}1, F, M_F) = -g'_\mu(m_e/m_\mu)\mu_B M_F B - \frac{hA_{\rm HFS}}{4} - (-1)^F \frac{hA_{\rm HFS}}{2}\sqrt{1 + 2M_F x + x^2}, \quad (53)$$

where

$$x = (g_J + g'_\mu(m_e/m_\mu))\mu_B B/(h\ A_{\rm HFS})\ \ ;\ \ x = 1 \Leftrightarrow B \approx 0.1585\ {\rm T} \quad (54)$$

is the magnetic field parameter. The Breit-Rabi equation does not incorporate the effect of higher quantum states. Such corrections of relative order $(\Delta\nu_{\rm HFS}/R_\infty)^2 \approx 10^{-12}$ are negligible. Off-diagonal elements of the Hamiltonian Equation (51) to higher n states would cause corrections of the size $(\mu_B B/R_\infty)^2$ and can be neglected for any magnetic field strength of practical interest.

5.1.2 Muonium formation in gases

Muonium atoms can be produced in reasonable quantities only at accelerator sites. The most efficient way of forming muonium is by stopping positive muons in suitable gases and by electron capture according to the reaction

$$\mu^+ + G \rightarrow M + G^+, \quad (55)$$

where G stands for the particular gas atom used. Chemical reactions and depolarisation effects can be avoided by using noble gases. Muonium can be principally formed in any quantum state. However, the process is only possible, if it is energetically allowed. Muonium has in its ground state a binding energy of $E_I(M) = 13.54$eV. For gases with higher ionisation potentials $E_I(G)$ the energy defect has to be brought up from kinetic energy E_t. The energy difference $E_I(G) - E_I(M)$ in the muonium-gas center of mass system transforms for room temperature gases with atom masses m_G with negligible kinetic energies into the laboratory frame as a threshold energy of

$$E_t = (E_I(G) - E_I(M))(m_\mu + m_G)/m_G. \quad (56)$$

For Xe we have $E_t(Xe) = -1.41$eV and muonium formation is always possible. Experimentally a muonium formation fraction of 100% is found (Stambaugh 1974). Of practical importance are the cases for Argon ($E_t(Ar) = 2.22$eV) and Krypton ($E_t(Kr) = 0.46$eV) with formation fractions of 65(5)% (Stambaugh 1974) and 80(10)% (Schwarz and Hughes 1993).

The slowing down of the muons and the electron capture process are mainly due to Coulomb interaction. Magnetic interactions are of negligible importance. Therefore the muon polarisation is not destroyed. The formation probabilities f_i $(i = 1, 2, 3, 4)$ of the ground state sublevels have been worked out using the density matrix formalism (Thompson 1973). The index i refers to the four different states for which the assignment of (F, M_F) quantum numbers at low fields and of (M_e, M_μ) quantum numbers in strong fields is shown in Figure 23. For a muon polarisation P in the z direction and for unpolarised electrons in the target material one finds in a magnetic field along the z axis

$$f_1 = \tfrac{1}{4}(1 + P),\ \ f_2 = \frac{1}{4}\left(1 + P(s^2 - c^2)\right),$$
$$f_3 = \tfrac{1}{4}(1 - P),\ \ f_4 = \frac{1}{4}\left(1 + P(c^2 - s^2)\right), \quad (57)$$

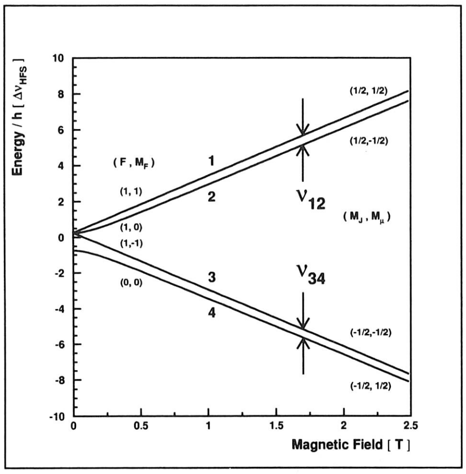

Figure 23. *Muonium ground state Zeeman levels in an external magnetic field. The two transitions indicated were measured in the latest experiments.*

where

$$s = \sin(\frac{1}{2}\text{arc cot}x) \quad \text{and} \quad c = \cos(\frac{1}{2}\text{arc cot}x) \tag{58}$$

with the dimensionless field parameter x as defined in Equation (54).

5.1.3 Experiments in magnetic fields

It is a consequence of the Breit-Rabi equation, Equation (53), that with the labeling of levels as shown in Figure 23 we have in magnetic fields

$$\nu_{12} + \nu_{34} = \Delta\nu^M_{\text{HFS}} \tag{59}$$

$$\nu_{12} - \nu_{34} = \frac{2\mu_\mu g'_\mu B}{h} + \Delta\nu^M_{\text{HFS}}\left[(1+x^2)^{1/2} - x\right]. \tag{60}$$

The sum of the frequency ν_{12} for a transition between level 1 and 2 and the frequency ν_{34} for a transition between level 3 and 4 is always equal to the $\Delta\nu^M_{\text{HFS}}$, the hyperfine splitting at zero fields. From the difference frequency one can obtain a value for the magnetic moment of the positive muon μ_{μ^+}. In an experimental situation one can assume similar absolute experimental errors for the transition frequency determination and constant relative experimental precision for the magnetic field at low and high magnetic field values. Therefore, it appears that high field measurements are better suited for measurements of the magnetic moment, whereas $\Delta\nu^M_{\text{HFS}}$ can be measured in both field regimes accurately.

In a series of experiments both strong and weak field experiments with steadily increasing accuracy (Prepost *et al.* 1961, Cleland *et al.* 1964, Thompson *et al.* 1969, Ehrlich *et al.*1969, DeVoe *et al.* 1970, Favart *et al.* 1971, Favart *et al.* 1973, Casperson *et al.* 1975, Casperson *et al.* 1977, Mariam *et al.* 1982) have been carried out. The latest and most accurate strong field experiments were carried out at LAMPF (Mariam *et al.* 1982, Liu *et al.* 1998) in Kr gas at pressures around one atmosphere. In order to be able to correct

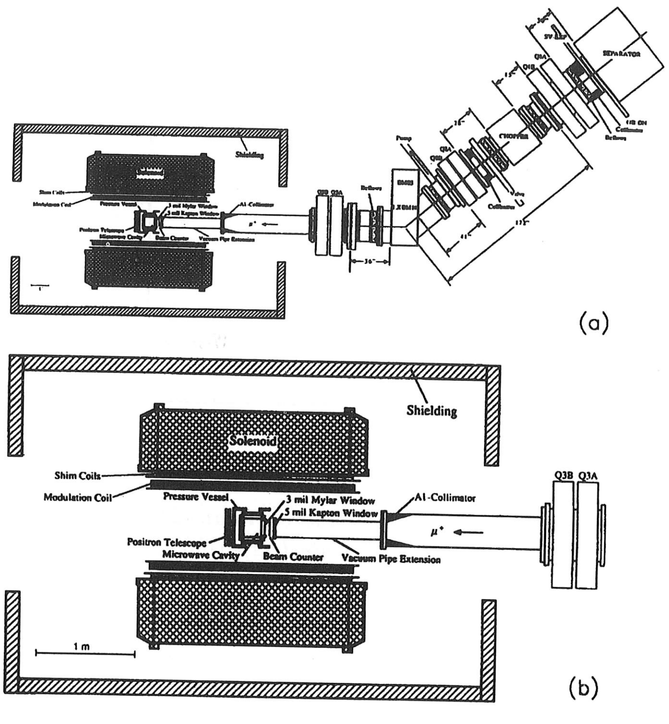

Figure 24. *The LAMPF muonium ground state hyperfine structure experiment. (a) An electrostatic chopping device in the beam line provides the pulses necessary for the old muonium technique. (b) Polarised muons are stopped in Kr gas inside of a microwave resonator located in a strong magnetic field. The microwave transitions are observed as a change in the spatial asymmetry of the decay positrons by the scintillation counters. (Mariam* et al. *1982).*

for density shifts in the Kr gas, data were recorder for different pressures between 0.5 and 1.5atm. The values corresponding to zero pressure were found by extrapolation. Polarised surface muons are stopped inside of a microwave resonator in the gas target, where they form polarised muonium in a strong magnetic field. According to Equation (57) the atoms are predominantly in the states ($M_e = +\frac{1}{2}, M_\mu = s$) and ($M_e = -\frac{1}{2}, M_\mu = s$), where s is either $+\frac{1}{2}$ or $-\frac{1}{2}$ depending on the relative orientation of the muon spin direction and the **B** field.

The direction of the muon spin can be detected by the spatial anisotropy of the decay positron distribution from the muon decay $\mu^+ \rightarrow e^+ + \overline{\nu}_\mu + \nu_e$ (Garwin *et al.* 1957, Friedman and Telegdi 1957). The microwave transitions ν_{12} and ν_{34} involve a change of the muon spin orientation which can be detected as a change of the spatial forward/backward asymmetry of the decay positrons in scintillation counters. A precision experiment in $B \approx 1.36$T field was completed in 1982 at LAMPF and found $\Delta\nu^M_{\rm HFS}$ = 4 463 302.88(16)kHz for the muonium ground state hyperfine splitting and μ_μ/μ_p = 3.183 346 1(11) for the ratio of the muon magnetic moment to the proton's magnetic moment. This can be converted into a value for the muon mass m_μ using m_e from (Caso 1998) $m_\mu = m_e(g_\mu/2)(\mu_\mu/\mu_p)(\mu_p/\mu_B) = 105.658\,386(44)\mathrm{MeV}/c^2$. With confidence in the QED calculations as quoted in Section 4.1.3 one can extract from the measurements a precise value for the fine structure constant α. A most recent calculation yields $\alpha^{-1}(M, HFS) = 137.035994(18)$ (Kinoshita 1995).

Experimental Method	Value of $\alpha^{-}1$	Accuracy	Reference
Muonium hyperfine structure	137.036 003 4(48)	36ppb	(Liu *et al.* 1998)
Electron g-2	137.035 999 93(52)	3.8ppb	(Kinoshita 1996)
Quantum Hall effect	137.036 003 7(33)	24ppb	(Jeffrey *et al.* 1997)
ac-Josephson effect	137.035 977 0(77)	56ppb	(Williams *et al.* 1989)
neutron de Broglie wavelength and R_∞	137.030 011 4(50)	37ppb	(Nistler 1998).

Table 6. *The fine structure constant α can be extracted from different measurements in atomic physics and in condensed matter.*

A new and recently completed experiment used an homogeneous magnetic field of 1.7Tesla. The experiment employed the technique of "old muonium" which allowed the reduction of the linewidth of the signals can be reduced below half of the "natural" linewidth $\delta\nu_{\mathrm{nat}} = (\pi \cdot \tau_\mu)^{-1}$=145kHz, where τ_μ is the muon lifetime of 2.2μs. For this purpose the basically continuous beam of the LAMPF stopped muon channel was chopped by an electrostatic kicker device into 4μs long pulses with 14μs separation. Only atoms which were interacting with the microwave field for periods longer than several muon lifetimes were detected (Boshier *et al.* 1995). Although less particles contribute to the signal, there is a gain in accuracy, as for short times, when most muons decay, only a few have undergone a transition. Depending on details of the experiment the is an optimum waiting time around 4-6μs. The basically statistically limited results improve the knowledge of both zero field hyperfine splitting and muon magnetic moment by a factor of three (Liu em et al. 1998).

Two directly measured frequencies at LAMPF experiment in 1.7T magnetic field are $\Delta\nu_{12} = 1\,897\,539\,791(35)Hz(18ppb)$ and $\Delta\nu_{34} = 2\,565\,762\,973(43)Hz(17ppb)$. This yields

$$\Delta\nu_{\mathrm{HFS}} = 4\,463\,302\,764(54)\mathrm{Hz}\ (\ 12\mathrm{ppb}) \tag{61}$$

in good agreement with the theoretical value of Equation (50). For the magnetic moment one finds

$$\mu_\mu/\mu_p = 3.183\,345\,26(39)(120\mathrm{ppb}) \tag{62}$$

which translates into a muon-electron mass ratio of $m_\mu/m_e = 206.768\,270(24)(120\mathrm{ppb})$.

As the hyperfine splitting is proportional to the fourth power of the fine structure constant α, the improvement in α will be much better than for $\Delta\nu_{HFS}$ and the value is comparable in accuracy with the value of α determined with the Quantum Hall effect (Jeffrey *et al.* 1997). Here the value $\hbar/m_e$ as determined with the help of measurements of the neutron de Broglie wavelength (Krüger *et al.*, 1995, Nistler 1998). can be beneficially employed to gain higher accuracy compared to the traditionally used determination based on the very well known Rydberg constant. In future this quantity may be obtained very accurately through photon recoil measurements under way at various atomic physics

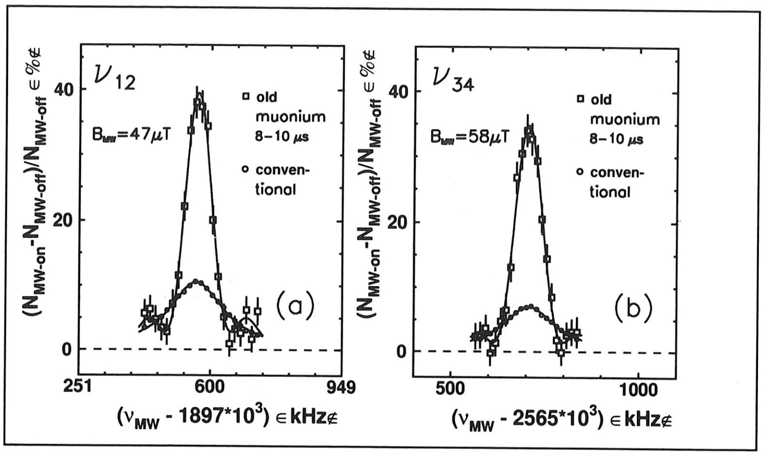

Figure 25. *Conventional and 'old' muonium lines. The narrow 'old' lines are also higher. The frequencies ν_{12} and ν_{34} correspond to transitions between (a) the two energetically highest respectively (b) two lowest Zeeman sublevels of the n =1 ground state. As a consequence of the Breit-Rabi equation describing the behaviour of the levels in a magnetic field, the sum of these frequencies equals at any fixed field the zero field splitting $\Delta\nu$ and the difference yields for a known field the muon magnetic moment m_μ.*

laboratories. At present the fine structure constant from muonium hyperfine splitting from Equation (34) is

$$\alpha^{-1} = 137.036\,010\,8(52)(39\text{ppb}). \qquad (63)$$

A traditional derivation from Equation (32) gives $\alpha^{-1} = 137.035\,998\,6(80)(59)$ppb.

The two standard deviation agreement of α values determined from the electron magnetic anomaly and from the hyperfine splitting in the muonium atom may be interpreted as the most precise reassurance of the internal consistency of QED, as the first case involves QED of free particles whereas in the second case distinctively different bound state QED approaches need to be applied (Kinoshita and Yennie 1990).

The limitation of α from muonium HFS arises mainly from to the muon mass. Therefore any better determination of the muon mass, respectively its magnetic moment, e.g. through a precise measurement of the reduced mass shift in the muonium 1s-2s splitting, will result in an improvement of this value of α. It should be noted explicitly that already at present the good agreement between the fine structure constant determined from muonium hyperfine structure and the one from the electron magnetic anomaly is considered the best test of internal consistency of QED, as one case involves bound state QED and the other case free particles.

It should be mentioned that a factor of five improvement both in the hyperfine splitting of the ground state and of the muon magnetic moment, which would lead to a well improved fine structure constant value from electromagnetically bound states, appears possible at present at PSI. With a dispersion corrected (see Eaton 1998) electrostatic chopper in the πE5 area one can expect muon rates of order 5-7$\times 10^7 \mu^+$/s in a chopped beam with a 1:3 on/off ratio. This is one order of magnitude above the particle flux at the LAMPF stopped muon channel with the additional benefit of a better beam spot.

5.2 Classical Lamb shift in muonium

5.2.1 Beam foil production of muonium

Muonium in vacuum can be obtained from a beam neutralisation technique analogous to methods in proton beam foil spectroscopy (Phillips 1955, Berry 1977). Positive muons passing through metal foils in vacuum may capture to some fraction an electron thus forming muonium (Ahn 1992, Bolton *et al.* 1981), and negative muonium ions (Kuang *et al.* 1987). The electron capture depends on the velocity, but not on the mass of the projectile particle (Kreussler and and Sizmann 1982). The rate is maximal for muon velocities equal to the Fermi energy of the electrons in the metal which is of the order of αc. There is no strong dependence on the particular material of the metal foil observed (Kettell 1990). For optimising the muonium rate one has to take into account that due to multiple scattering in the muon production target the beam rates are proportional to the 3.5th power of the beam momentum. In addition, at low energies losses due to muon decay in flight become important. Therefore the optimal muonium production momentum is about 7.3MeV/c at which a yield of 12% muonium atoms per incident muon was observed at LAMPF for a 200μg/cm^2 Al foil (Kettell 1990).

The muonium atoms have typical kinetic energies of 5keV. The angular distribution has been found to be dominated by multiple scattering of the muon in the target foil (Ahn 1992). A fraction of $\approx$ 10% of the atoms is in the metastable $2^2S_{1/2}$ state (Kettell 1990) in accordance with a $1/n^3$ law for the distribution between quantum states as observed for atomic hydrogen (Gabrielse 1981). The negative muonium ion ($\mu^+e^-e^-$) has been observed with a production yield of 1.5×10^{-5} per incident muon.

5.2.2 Classical $2S_{1/2} - 2P_{1/2}$ Lamb shift

In independent experiments at TRIUMF (Oram *et al.* 1984) and LAMPF (Badertscher *et al.* 1984, 1990, Kettell 1990) both the $2^2S_{1/2} - 2^2P_{1/2}$ classical Lamb shift transitions and $2^2S_{1/2} - 2^2P_{3/2}$ fine structure transitions could be induced. Beam foil produced fast muonium atoms in the metastable $2^2S_{1/2}$ state were passed through a section of a rf transmission line ($2^2S_{1/2} - 2^2P_{1/2}$) respectively a tuneable microwave cavity ($2^2S_{1/2} - 2^2P_{3/2}$). A transition to a P state is followed by the rapid decay into the ground state within the natural P state lifetime of τ_P=1.6ns. The occurrence of a transition was observed as a reduction of the $2^2S_{1/2}$ population which was probed further downstream by quenching the $2^2S_{1/2}$ state through admixing of the close lying short lived $2^2P_{1/2}$ state in an external electrostatic field and by observing Lyman-α radiation in CsI coated microchannel plates (TRIUMF) or UV sensitive photomultipliers (LAMPF). For small electric fields E (Stark energy small compared to the fine structure splitting, i.e. $E <$ 3.5kV/cm), the lifetime of the $2^2S_{1/2}$ state is shortened to (Bethe and Saltpeper 1957)

$$\tau_{2S} \simeq \tau_{2P}\left[1+\left(\frac{475\ \mathrm{V/cm}}{E}\right)^2\right]. \tag{64}$$

The apparatus of the TRIUMF experiment is shown in Figure 26 together with the signals obtained. The hyperfine structure is resolved.

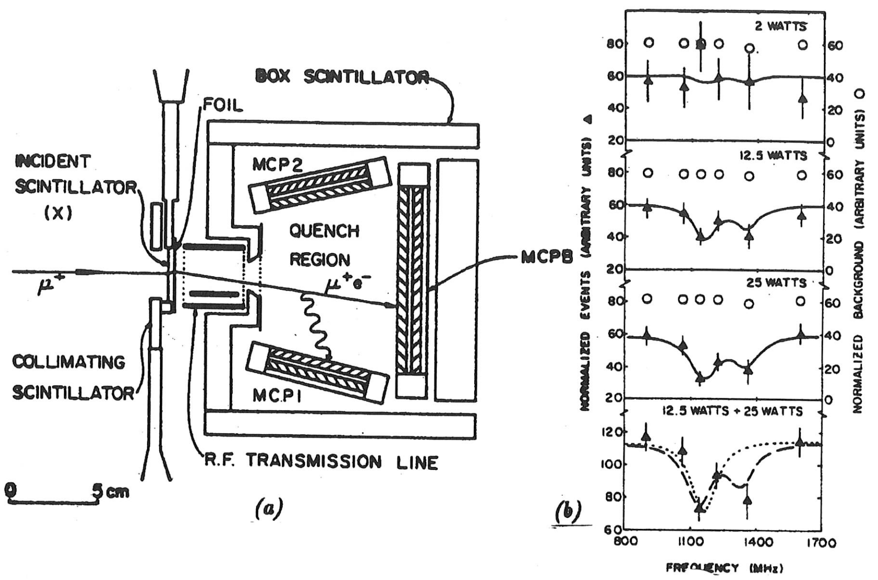

Figure 26. *Measurement of the muonium n =2 Lamb shift at TRIUMF (*Oram et al. *1984). (a) A schematic view of the experimental setup. Fast metastable 2S muonium is produced by a beam foil technique and Lamb shift transitions are induced in a rf transmission line. The 2S population is probed by an electric quench field and Lyman-α photon detection. (b) Signals at different power levels.*

Experiment	$\Delta\nu_{2^2S_{1/2}-2^2P_{1/2}}$	experimental method	Ref.
TRIUMF	1 070(+12)(-15)MHz	direct $2^2S_{1/2}-2^2P_{1/2}$ trans.	Oram *et al.* 1984
LAMPF	1 042(+21)(-23)MHz	direct $2^2S_{1/2}-2^2P_{1/2}$ trans.	Badertscher *et al.* 1990
LAMPF	1 027(+30)(-35)MHz	from $2^2S_{1/2}-2^2P_{3/2}$ trans.	Kettell 1990
Theory	1 047.49(1)(9)MHz	–	Sapirstein and Yennie 1990

Table 7. *Classical $2^2S_{1/2}-2^2P_{1/2}$ Lamb shift in muonium. Comparison between experiment and theory.*

The fine structure measurement at LAMPF (Kettell 1990) has yielded a $2^2S_{1/2}-2^2P_{3/2}$ transition frequency of $9\,895^{+35}_{-30}$MHz which can be converted into a Lamb shift value by subtracting it from the calculated $2^2P_{1/2}-2^2P_{3/2}$ energy difference of 10 921.833(3)MHz (Lepage *et al.* 1984).

The results for the classical Lamb shift in muonium are listed in Table 7. In the experiments an accuracy of 1.4% could be reached. They are in good agreement with QED calculations (Sapirstein and Yennie 1990). But they are not yet an as severe test for theory as the measurements in hydrogen, where the classical Lamb shift is measured in a fast beam to be $\Delta\nu^H_{2^2S_{1/2}-2^2P_{1/2}}$(exp)=1 057.845(9)MHz (Lundeen and Pipkin 1986, Pipkin 1990) and in an atomic interferometer to be $\Delta\nu^H_{2S_{1/2}-2P_{1/2}}(exp)$=1 057.851 4(19)MHz

(Palchikov *et al.*1985) in agreement with the theory which finds $\Delta\nu^H_{2S_{1/2}-2P_{1/2}}(theor) = 1\,057.873(11)$MHz (Yennie 1992). Significant differences other than reduced mass effects between hydrogen and muonium are caused by the nuclear size effect in hydrogen of 145(4)MHz. The differences in the QED contributions are due to radiative recoil (≈+24.7kHz) and higher order recoil (≈-19.5kHz). This is new and untested physics which requires for an experimental accuracy of several kHz.

5.3 Ground state hyperfine structure splitting in vacuum

The precision measurements of the muonium hyperfine splitting performed so far suffer from the interaction of the muonium atoms with the foreign gas atoms. With the discov-

material	density [mg/cm^3]	thickness [mg/cm^2]	opt. muonium fraction $\mu^+e^-/\mu_{\rm stop}$ [%]	Ref.
SiO_2 powder	32	4.6	17(1)	(Woodle *et al.* 1988)
SiO_2 powder	32	2.8	15.9(3.6)	(Janissen *et al.* 1990)
SiO_2 powder	32	9.0	8.27(31)	(Springer 1993)
SiO_2 aerogel	5	7.5	2.32(13)	(Springer 1993)
SiO_2 aerogel	18	9	1.57(20)	(Springer 1993)
W foil (2130K)	19.3	96.5	4(2)	(Mills *et al.* 1986)
$C_{60/70}$ fullerens	≈ 1400	≈ 210	1.85(23)	(Springer 1993)
Cotton	10	3.6	2.25(46)	(Springer 1993)
Cotton coated with SiO_2 powder	17	5.8	11.43(31)	(Springer 1993)
Microchannel plate	≈ 2000	≈ 400	2.44(34)	(Springer 1993)

Table 8. *Thermal muonium in vacuum production. Different methods and target materials are compared which produce thermal vacuum muonium with significantly different efficiencies. The beam momentum has been optimised for maximum effect.* SiO_2 *powder targets are evidently at present the optimal choice as converter material.*

ery of polarised thermal muonium emerging from SiO_2 powder targets and other materials into vacuum (see Table 8) (Woodle *et al.* 1988, Janissen *et al.* 1990) measurements of the $\Delta\nu_{\rm HFS}$ are now feasible in vacuum in the absence of a perturbing foreign gas. Corrections for density effects are obsolete.

In a pioneering experiment at PSI the transitions between the $1^2S_{1/2}, F = 1$ and the $1^2S_{1/2}, F = 0$ hyperfine levels could be induced in zero magnetic field (see Figure 1). The atoms were formed by electron capture after stopping positive muons close to the surface of a SiO_2 powder target. A fraction of these diffused to the surface and left the powder at thermal velocities (7.43(2)mm/μs) for the adjacent vacuum region. A rectangular rf resonator operated in TEM_{301} mode was placed directly above the muonium production target. The atoms entered the cavity with thermal velocities through a wall opening. The e^+ from the parity violating muon decay $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$ were registered in two scintillator telescopes which were mounted close to the side walls of the resonator. The telescope axes have been oriented parallel respectively antiparallel to the spin of the incoming muons which had been rotated transverse to the muon propagation direction by an **E**×**B** separator (Wien filter) in the muon beam line. At the resonance frequency

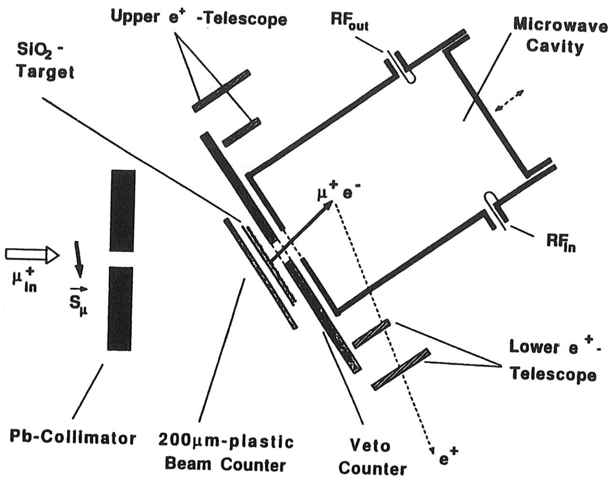

Figure 27. *Principle of the setup for a measurement of $\Delta\nu_{\rm HFS}$ in vacuum.*

of 4.46329(3)GHz a reduction of the muon polarisation of 16(2)% has been observed as a signal from the hyperfine transitions (Jungmann K *et al.* 1995).

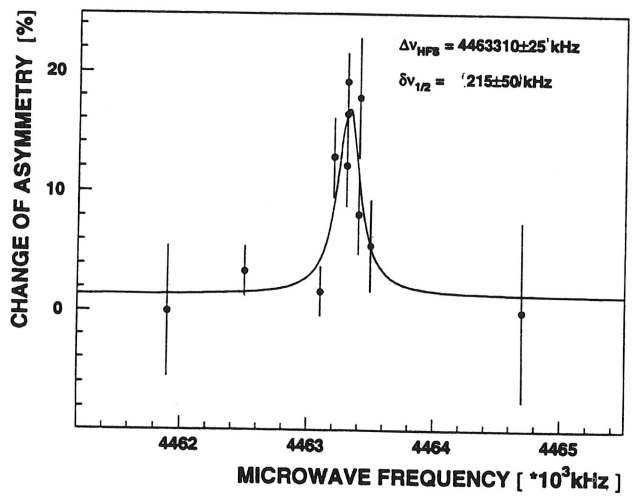

Figure 28. *First observed muonium $\Delta\nu_{\rm HFS}$ signal in vacuum.*

Certainly the method needs a lot of development work in order to improve the signal strength. However, ultimately one expects higher precision from experiments on muonium in vacuum than from measurements in gases.

6 Two-photon spectroscopy of muonium

The $1^2S_{1/2}$ and the $2^2S_{1/2}$ states in hydrogen-like systems are both of odd parity. Single photon electric dipole transitions are parity forbidden. Two-photon transitions (Göppert M-Mayer 1931, Vasilenko 1970, Smith and Hänsch 1971, Hänsch 1977, Lethokhov and Chebotaev 1977, Giacobino and Cagnac 1980), however, are allowed. Indeed, they are the dominant radiative decay channel for the metastable 2S state leading to a lifetime of τ_{2S}=1/7s (Bethe and Salpeper 1957) in natural hydrogen.

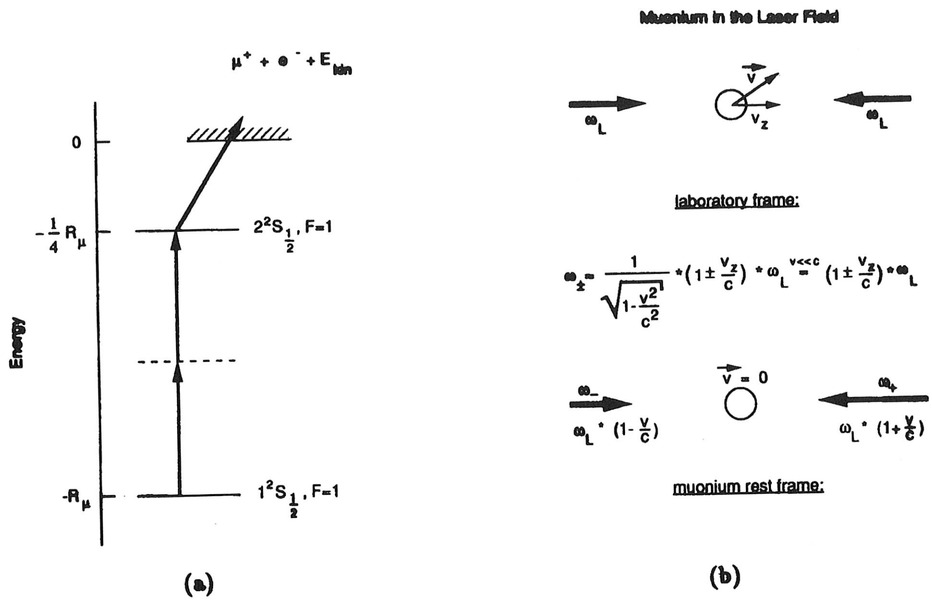

Figure 29. *Principle of the muonium 1S-2S experiment. (a) The transition between the $1^2S_{1/2}$ and the $2^2S_{1/2}$ state is induced by the simultaneous absorption of two photons from two counter-propagating laser beams at $\lambda = 244.2nm$ wavelength. The metastable 2S state is photoionised by a third photon from the same light field and the muon set free by the ionisation is detected as a signal for the two-photon transition. (b) The transition is Doppler-free to first order for the absorption of two photons from each of the beams.*

Two-photon transitions are second order in perturbation theory. The strength of an absorption signal for two photons of equal energy depends strongly on the level structure of the atom under investigation. As a general rule, however, one needs very high intensities to achieve significant transition probabilities. Such intensities are available from pulsed laser sources only. Since muonium atoms can be produced at moderate quantities (as compared to natural hydrogen which is available from gas bottles) an experiment benefits largely from a pulsed accelerator muon source. In addition, it is of importance for experiments that accidental background in the detectors can be suppressed in a pulsed experiment by suitable gating. The two-photon transition is Doppler-free to first order (Vasilenko *et al.* 1970), if it is induced by the absorption of one photon from each of two counter-propagating laser beams, because a moving atom sees the two beams Doppler shifted with opposite sign and to first order by the same amount (see fig. 29).

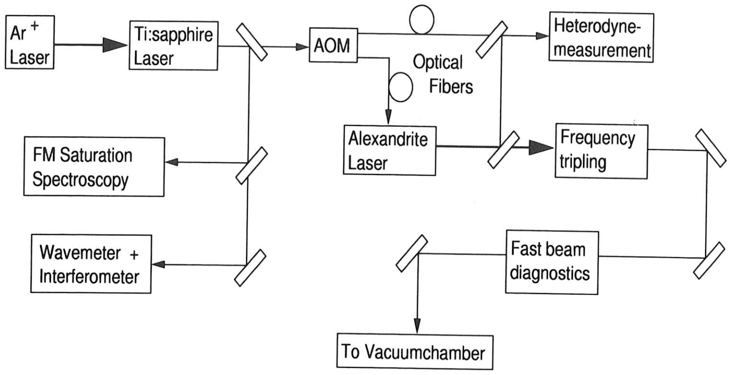

Figure 30. *Laser system employed in the new muonium 1s-2s experiment.*

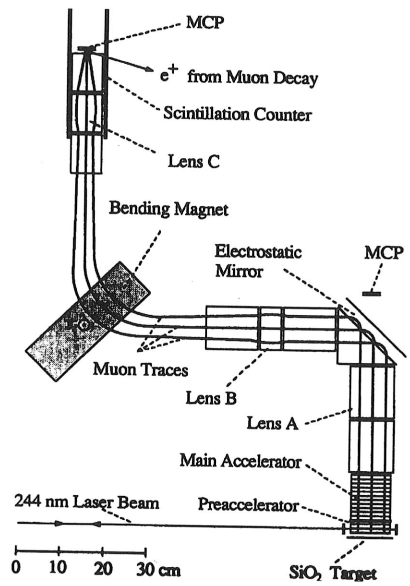

Figure 31. *Schematics of the detector setup for the RAL muonium 1s-2s laser spectroscopy experiment.*

Doppler free excitation of the 1s-2s transition in muonium has been achieved in the past at KEK in Tsukuba, Japan, (Chu *et al.* 1988) and at the Rutherford Appleton Laboratory (RAL) in Chilton, Didcot, UK (Jungmann *et al.* 1991, Maas *et al.* 1994). The accuracy of the last experiment was limited by ac Stark effect (the effect of the strong electric field in the laser beams on the atomic level structure) and a frequency chirp (time dependent laser frequency variation) caused by rapid changes of the index of refraction in the dye solutions of the laser amplifier employed in the laser system. The experiment yielded $\Delta\nu_{1S-2S} = 2\,455\,529\,002\ (33)(46)$MH z for the centroid (corrected for hyperfine structure) 1S-2S transition frequency. This value is in agreement with theory within two standard deviations (Yennie 1992). The Lamb shift contribution to the 1S-2S splitting has been extracted to $\Delta\nu_{LS} = 6\,988\ (33)(46)$MHz; this the most precise experimental Lamb

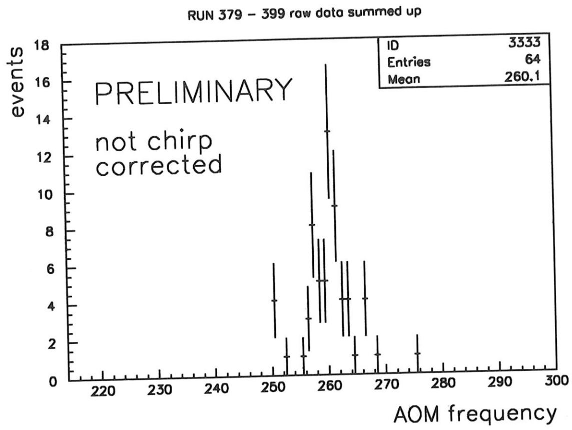

Figure 32. *The $1^2S_{1/2}(F{=}1) \rightarrow 2^2S_{1/2}(F{=}1)$ transition signal in muonium, not corrected for systematic shifts due to frequency chirping and ac Stark effect.*

shift value for muonium available today. From the isotope shifts between the muonium signal and the hydrogen and deuterium 1S-2S two-photon resonances we deduce the mass of the positive muon as $m_\mu = 105.658\,80\ (29)(43)\mathrm{MeV/c^2}$. An alternate interpretation of the result yields the best test of the equality of the absolute value of charge units in the first two generations of particles at the 10^{-8} level (Maas *et al.* 1994).

A new measurement of the 1S-2S energy splitting of muonium by Doppler-free two-photon spectroscopy has been performed at RAL where the world's brightest pulsed surface muon channel exists. At RAL protons are accelerated in the ISIS synchrotron to 800MeV. The extracted proton beam shows a double pulse structure with 330ns separation between pulses and has a repetition rate of 50Hz. At a muon beam momentum of 26.5MeV/c the DEVA port of the channel delivers about 3000 μ^+ to the experiment in a 10% momentum band with almost 100% polarisation, if a 5mm carbon target is used for pion production (Eaton *et al.* 1988). An $E \times B$ separator in the beam line (Wien filter) reduces positron background efficiently to below 1.5%. The muon beam reflects the time pulse structure of the primary proton beam with a width of 82ns for every single pulse component.

For the experiment thermal muonium atoms in vacuum are produced by the SiO_2-powder technique (see Section 5.3) (Woodle *et al.* 1988) The muonium production was optimised and monitored by observing the decay positrons from the decay $\mu^+ \rightarrow e^+ + \overline{\nu}_\mu + \nu_e$ in a wire chamber telescope. The equipment has been optimised for the high instantaneous decay positrons rates after stopping a muon pulse in the target. More than one positron passes on average through the telescope within a muon lifetime τ_μ. Fast wire chamber readout electronics has been especially developed which is based on VME multi-hit TDC modules. The muonium atoms interact close above the target ($\approx$ 8mm) with two counterpropagating high power laser beams at $\lambda = 244.2$nm wavelength. The pulsed laser light arrives $t_L = 1.8\,\mu$s after the muons in the interaction region. It has a diameter of $2r_L = 2.5(5)$mm and propagates in d_L=8(1)mm distance parallel to the target. The laser beam crosses the muonium cloud 10 times between two mirrors and is then reflected back onto itself in order to enhance the overlap of the cloud and the laser light. Numerical estimates under the assumption of a $\cos\Theta$ angular distribution, where Θ

is the angle between the atom velocity and the powder surface normal direction, and the Maxwell-Boltzmann velocity distribution yield about 1 atom in the laser beam for every laser pulse.

The necessary high power UV laser light was generated in a newly developed laser system by frequency tripling the output of an alexandrite ring laser amplifier in crystals of LBO and BBO. Typically UV light pulses of energy 3mJ and 80nsec (FWHM) duration were used. The alexandrite laser was seeded with light from a continuous wave Ti:sapphire laser at 732nm which was pumped by an Ar ion laser. Fluctuations of the optical phase during the laser pulse were compensated with an electro-optic device in the resonator of the ring amplifier to give a frequency chirping of the laser light of less than about 5MHz. The laser frequency was calibrated by frequency modulation saturation spectroscopy of a hyperfine component of the 5-13 R(26) line in thermally excited iodine vapour. The frequency of the reference line is about 700MHz lower than 1/6 of the muonium transition frequency. The cw light was frequency up-shifted by passing through two acousto-optic modulators (AOM's). The muonium reference line has been calibrated preliminarily to 3.4MHz at the Institute of Laser Physics in Novosibirsk. An independent calibration at the National Physics Laboratory (NPL) at Teddington, UK, was performed to 140kHz (0.35ppb) (Baird *et al.* 1991).

The 1S-2S transition was detected by the photoionisation of the 2S state by a third photon from the same laser field. The slow muon set free in the ionisation process is accelerated to 2keV and guided through a momentum and energy selective path onto a microchannel plate particle detector (MCP). Background due to scattered photons and other ionised particles can be reduced to less than 1 event in 5 hours by shielding and by requiring that the MCP count falls into a 100nsec wide window centered at the expected time of flight for muons and by additionally requiring the observation of the energetic positrons from the muon decay. On resonance an event rate of 9 per hour was observed. The number of MCP events as a function of the laser frequency is displayed in Figure 6.

The line shape distortions due to frequency chirping were investigated theoretically using density matrix formalism to find numerically the frequency dependence of the signal for the time dependent laser intensity and the time dependent phase of the laser field which were both measured for every individual laser shot using fast digitized amplitude and optical heterodyne signals (Yakhontov and Jungmann 1996, Yakhontov *et al.* 1998). The model was verified experimentally at the ppb level by observing resonances in deuterium and hydrogen in the same apparatus. A careful analysis is in progress. Preliminary results indicate that the accuracy will be of order 10MHz for the 1s-2s level splitting.

It can be expected that future experiments will reach well below 1MHz in accuracy promising an improved muon mass value. The theoretical value at present is basically limited by recoil terms at the 0.6MHz level. Even cw laser excitation, which would totally avoid any chirping problems, may be possible at future high intensity muon sources under construction or in planning (see 7).

At KEK a group was successful in producing a beam of slow muons, i.e. muons of energies in the eV region, by first exciting muonium into the 2P state and the by photoionisation with a second laser (Nagamine *et al.* 1995). This may open a path leading to ultra slow muon beams for studies of thin films or condensed matter surfaces.

7 Long term future possibilities

It appears that the availability of particles limits the ability to derive much better values for fundamental constants, to find nonstandard physical processes or to impose significantly improved limits in continuation of the search programs of existing dedicated experiments. Therefore any measure to boost the particle fluxes is a very important step forward. The $\pi - \mu$ converter at PSI or the dedicated tailored muon production of the planned MECO experiment at BNL are examples of novel attempts to overcome this problem. In principle, we need significantly more intense accelerators, such as they are presently discussed at various places. For some experiments, like improvements in the muon lifetime, studies of muon capture in light nuclei or precision muonium spectroscopy, one could have very valuable improvements on a short time scale from an intense chopped muon beam. Such a facility would be possible at PSI in the πE3 or even better in the πE5 beam area with some well designed chopping device added to the existing facilities (Kammel *et al.* 1998). For the mentioned experiments a pulsed beam structure is already at present more important than highest integral particle fluxes. In the intermediate future the Japanese Hadron Facility (JHF) or a possible European Spallation Source (ESS) are important options. Also the discussed Oak Ridge neutron spallation source could in principle accommodate intense muon beams. The most promising facility would be, however, a muon collider (Palmer and Gallardo 1998), the front end of which could provide muon rates 5-6 orders of magnitude higher than present beams (see Table 9).

	RAL(μ^+)	PSI(μ^+)	PSI(μ^-)	JHF(μ^+)†	ESS(μ^+)	MC (μ^+, μ^-)
Intensity (μ/s)	3×10^6	3×10^8	1×10^8	4.5×10^7	4.5×10^7	7.5×10^{13}
Momentum bite						
Δ pm/p[%]	10	10	10	10	10	5-10
Spot size						
Spot size (cm×cm)	1.2×2.0	3.3×2.0	3.3×2.0	1.5×2.0	1.5×2.0	few×few
Pulse structure	82ns	50MHz	50MHz	300ns	300ns	50ps
	50Hz	cont.	cont.	50Hz	50Hz	15Hz

Table 9. *Muon fluxes of some existing and future facilities, Rutherford Appleton Laboratory (RAL), Japanese Hadron Facility (JHF), European Spallation Source (ESS), Muon collider (MC). (†Recent studies indicate that the 10^{11} particles/s region might be reachable (Kuno 1998).*

At such facilities low and high energy experiments come close together. It was noted, for example, already in the early sixties that, e.g. the process $e^-e^- \to \mu^-\mu^-$ is closely related to muonium-antimuonium conversion (Glashow 1961). Indeed such scattering experiments were carried out at the Princeton-Stanford storage rings at Stanford yielding the at the time best limit on the coupling constant $G_{\mathrm{M\overline{M}}}$ (Barber *et al.* 1969). Today, similar proposals have been made for scattering of high energy e^- on e^-, e^- on e^+, μ^- on μ^- and μ^- on μ^+ (Frampton 1992, Hou 1996, Raidal 1998). They were mainly discussed in connection with bileponic gauge bosons. Even a lower limit for the cross section of the process $e^-e^- \to \mu^-\mu^-$ was found, provided the sum of the light neutrino masses exceeds ≈ 90 eV (Raidal 1998). Pronounced resonances have been predicted particularly for such experiments at the Next Linear Collider or the high energy end of a muon collider.

Although the nature of the muon, the reason for its existence (as well as of other higher generation particles) remains a mystery, both the theoretical and experimental work in fundamental muon physics have contributed to a deeper understanding of particle interactions in general. One particular value of the experiments in the field are their continuos contributions towards guiding theoretical developments by excluding various speculative models.

It will remain a privilege of the future to find, maybe, an answer to Professor Rabi's important question, which he asked after learning about the leptonic nature of the muon:

"Who ordered that?"

Acknowledgments

It is a pleasure to thank the organisers of this muon summer school for the opportunity to include a few pieces of the rich and exciting field of fundamental muon physics. Part of the work reported was supported by the German Bundesminister für Bildung und Forschung (BMBF) and by a NATO research grant. The author is grateful to his collaborators on several muon and muonium experiments, who all contributed to increasing our knowledge about the muon.

References

Abela R *al.*, 1996, *Phys Rev Lett* **77** 1951.

Adriani O *et al.*, 1993, *Phys Lett B* **316**, 427.

Ahn H E,1992, PHD thesis, Yale University

Ammar R *et al.*, 1994, *Phys Rev D* **49**, 5701.

Assamagan K, Brönimann Ch, Daum M, Forrer H, Frosch R, Gheno P, Horisberger R, Janousch M, Kettle R P, Spirig Th and Wigger C, 1994, *Phys Lett B* **335**, 231.

Athanassopoulos C *et al.*, 1994, *Phys Rev C* 54, 2685; see also: nucl-ex/9709006 and nucl-ex/9709006

Auld E G, Averdung H, Bailey J M, Beer G A, Dreher B, Drumm H, Erdmann K, Gastaldi U, Klempt E, Merle K, Neubecker K, Sabev C, Schwenk H, Walter V H, Wendling R D, White B L, and Wodrich W R,1978, *Phys Lett* **77B**, 454.

Badertscher A, Dhawan S, Eagen P O, Hughes V W, Lu D C, Ritter M W, Woodle K A, Gladisch M, Orth H, zu Putlitz G, Eckhause M, Kane J, Mariam F G, and Reidy J, 1984, *Phys Rev Lett* **52**, 914.

Badertscher A, Hughes V W, Lu D C, Ritter M W, Gladisch M, Orth H, zu Putlitz G, Eckhause M, Kane J, and Mariam F G, 1990, *Phys Rev A* **41**, 93.

Baird P, Barwood G P, Cornish S, Liu W, 1998, publication in preparation; see also: Barwood G P, Rowley W R C, Gill P, Flowers J L, and Petley B W, 1991, *Phys Rev A* **43**, 4783.

Baklalov D, Milotti E, Rizzo C, Vacchi A, and Zavattini E, 1993, *Phys Lett* **A 172** 277.

Barber W C *et al.*, 1969, *Phys Rev Lett* **22** 902.

Barbieri R, Hall L and Strumia A, 1995, *Nucl Phys B* **445** 219.

Bargmann V, Michel L and Telegdi V, 1959, *Phys Rev Lett* **2** 435.

Beer W, Bogdan M, Goudsmit P F A, Leisi H J, Rusi A J El Hassani, Sigg D, Thomann S, Volken W, Bovet D, Bovet E, Chatellard D, Egger J P, Fiorucci G, Gabathuler K, and Simons L M, 1991, *Phys Lett* **B261** 16.

Bég M A B and Feinberg G, 1974, *Phys Rev Lett* **33**, 606; and 1975, *Phys Rev Lett* **35**, 130.
Belz J, Proc Intersections between Particle and Nuclear Physics, 6th conf, editor Donnelly T W, (AIP Press, New York) 763; Ray R, 1998, JHF98 workshop, Tsukuba.
Bernabeu J, 1992, *Z Phys C* **56** S24.
Berry H G, 1977, *Rep Prog Phys* **40** 155.
Bertl W *et al.*, 1985, *Nucl Phys B* **260** 1.
Bethe H A and Salpeter E E, 1957, Quantum Mechanics of One- and Two- electron Atoms (Springer, Berlin Göttingen Heidelberg).
Bhatt G and Grotch H, 1985, *Phys Rev A* **31** 2794; 1987, *Phys Rev Lett* **58** 471; 1987, *Ann Phys (NY)* **178** 1.
Blundell S, Cheng K, Sapirstein J, 1997, *Phys Rev Lett* **78** 4914.
BNL proposal E821 and Hughes V W 1994, A Gift of Prophecy, editor Sudarshan E C G (World Scientific) 222.
Bolton P R, Badertscher A, Egan P O, Gardener C J, Gladisch M, Hughes V W, Lu D C, Ritter M, Souder P A, Vetter J, zu Putlitz G, Echhause M, and Kane J, 1981, *Phys Rev Lett* **47** 1441.
Bolton T *et al.*, 1988, *Phys Rev D* **38** 2077.
Boshier M G *et al.*, 1995, *Phys Rev A* **52** 1948.
Boshier M G, Hughes V W, Jungmann K and zu Putlitz G, 1996, *Comm At Mol Phys* **33** 17.
Breit G and R, Meyerott E, 1947, *Phys Rev* **71** 1023.
Brodsky S J and Drell S D, 1970, *Ann Rev Nucl Sci* **20** 147.
Bugge L *et al.*, 1996, in: Proc Europhysics Conference on High-energy Physics, Brussels, editors Lemonne J *et al.* (World Scientific, Singapore); Abreu P *et al.*, 1997, *Z Phys C* **73**, 243.
Cabbibo N and Gatto R, 1960, *Phys Rev Lett* **5** 114.
Cabbibo N, 1961, *Nuovo Cim* **19** 612.
Cage M E *et al.*, 1989, *IEEE Trans Instrum Meas* **38** 284.
Canter K F, Mills A P, and Berko S, 1974, *Phys Rev Lett* **33** 7.
Carboni G, Gastaldi U, Neri G, Pitzurra O, Polacco E, Torelli G, Bertin A, Gorini G, Placci A, Zavattini E, Vitale A, Duclos J, and Picard J, 1976, *Nuov Cim* **34A** 493.
Carboni G, Gorini G, Torelli G, Palffy L, Palmonari F, and Zavattini E, 1977, *Nucl Phys A* **278** 381.
Carey R *et al.*, 1998 BNL Letter of Intent; Roberts B L, 1998, (private communication.)
Caso C *et al.* (Particle Data Group), 1998, *European Physical Journal C* **3** 1.
Casperson D E, Crane T W, Denison A B, Egan P O, Hughes V W, Mariam F G, Orth H, Reist H W, Souder P A, Stambaugh R D, Thompson P A, and zu Putlitz G, 1977, *Phys Rev Lett* **38** 956.
Casperson D E, Crane T W, Hughes V W, Souder P A, Stambaugh R D, Thompson P A, Orth H, zu Putlitz G, Kaspar H F, Reist H W, and Denison A B, 1975, *Phys Lett* **59B** 397.
Chatellard D *et al.*, 1997, *Nucl Phys* **A625** 855; Sigg D *et al.*, 1996, *Nucl Phys* **A609**, 269; D Sigg *et al.*, 1995, *Phys Rev Lett* **75** 3245.
Chatterjee L *et al.*, 1992, *Phys Rev D* **46** 46.
Chattopadhyay U and Nath P, 1996, *Phys Rev D* **53** 1648.
Chu S, Mills A P Jr., and Hall J L, 1984, *Phys Rev Lett* **52** 1689.
Chu S, Mills A P Jr., Yodh A G, Nagamine K, Miyake Y, and Kuga T, 1988, *Phys Rev Lett* **60** 101.
Cleland W E, Bailey J M, Eckhause M, Hughes V W, Mobley R M, Prepost R, and Rothberg J E, 1964, *Phys Rev Lett* **13** 202.
Cohen E R and Taylor B N, 1987, *Rev Mod Phys* **59** 1121.

Coombes R, Flexer R, Hall A, Kennelly R, Kirkby J, Piccioni R, Porat D, Schwartz M, Spitzer R, Toraskar J, Wiesner S, Budick B, and Kast J W, 1977, Atomic Physics 5, editors Marrus R, Prior M, and Shugart H (Plenum New York) 95.
Cooper M D *et al.*, 1997, loc cit Belz, 34.
de Beauvoir B, Nez F, Julien L, Cagnac B, Biraben F, Touahri D, Hilico L, Acef O, Clairon A, and Zondy J J, 1997, *Phys Rev Lett* **78** 440.
Danzmann K, Fee M S, and Chu S, 1989, *Phys Rev A* **39** 6072.
Darwin C G, 1928, *Proc Roy Soc London A* **118**.
Daum M, Frosch R, Herter D, Janousch M, and Kettle P, 1991, *Phys Lett* **B265** 425.
Daum M, Frosch R, Herter D, Janousch M, and Kettle P, 1992, *Z Phys C* **56** 114.
Deutsch J, 1992, *Z Phys C* **56** 143.
Deutsch M, 1951, *Phys Rev* **82** 455.
Doncheski M, Grotch H and Erickson G W, 1991, *Phys Rev A* **437** 2152.
Eaton G H, Carne A, Cox S F J, Davies J D, de Renzi R, Hartmann O, Kratzer A, Ristori C, Scott C A, Sterling G C, and Sundquist T, 1988, *Nucl Instr and Meth A* **269** 483 and Eaton G H, 1992, *Z Phys C* **56** 243.
Eaton G H, Scottish Summer School on Physics 51, 1998, St Andrews.
Eckhause M, Guss P, Joyce D, Kane J R, Siegel R T, Vulcan W, Welsh R E, Whyley R, Dietlicher R, and Zehnder A, 1986, *Phys Rev A* **33** 1743.
Edwards K *et al.*, 1997, *Phys Rev D* **55** 3919.
Eggli S *et al.*, publication in preparation (1998)
Ehrlich R D, Hofer H, Magnon A, Stowell D Y, Swanson R A, and Telegdi V L, 1969, *Phys Rev Lett* **23** 513.
Eides M and Shelyuto T, 1995, *Phys Rev* **A 52** 954; Eides M I, Grotch H and Shelyuto V A, 1998, *Phys Rev D* **58**, 013008.
Eides M I, Grotch H, and Owen D A, 1992, *Phys Lett* **B294** 115; Eides M I and Grotch H, 1993, *Phys Lett* **B301**, 127 and 1993, *Phys Lett* **B308** 389; Eides M I and Shelyuto V A, 1995, *Phys Rev A* **52** 954.
Eides M I, 1996, *Phys Rev A* **53** 2953.
Eitel K, 1995, doctoral thesis, University of Karlsruhe.
Erickson G W and Grotch H, 1988, *Phys Rev Lett* **60** 2611 and 1989, *Phys Rev Lett* **63** 1326.
Erickson G W, 1977, *J Phys Chem Ref Data* **6** 833.
Ericson T E O *et al.*, 1984, *Nucl Phys A* **416** 281.
Essen L, Donaldson R W, Bangham M J, and Hope E G, 1971, *Nature* **229** 110.
Farley F J M and Picasso E, 1990, in: Quantum Electrodynamics, editor (Kinoshita T), (World Scientific, Singapore) 479.
Favart D, McIntyre P M, Stowell D Y, Telegdi V L, DeVoe R G, and Swanson R A, 1973, *Phys Rev A* **8** 1195.
Favart D, McIntyre P M, Stowell D Y, Telegdi V L, DeVoe R G, and Swanson R A, 1971, *Phys Rev Lett* **27** 1336.
Fee M S, Mills A P, Chu S, Shaw E D, Danzmann K, Chichester R J, and Zuckermann D, 1993, *Phys Rev Lett* **70** 1397.
Feinberg G and Weinberg S, 1961, *Phys Rev Lett* **6** 381.
Feinberg G and Weinberg S, 1961, *Phys Rev* **123** 1439.
Fell R N, 1992, *Phys Rev Lett* **68** 25.
Fermi E, 1930, *Z Phys* **60** 320.
Flegel W and Krienen F, 1973, *Nucl Instr Meth* **113** 549.
Frampton P and Harada S, 1997, hep-ph/9711448.
Frampton P, 1998, private communication.
Frampton P, 1992, *Phys Rev D* **45** 4240.

Frampton P, 1994, *Phys Rev Lett* **69** 1889; see also: hep-ph/97112821.
Freyberger A *et al.*, 1996, *Phys Rev Lett* **76** 3065.
Fricke G *et al.*, 1995, Atomic Data and *Nuclear Data Tables* **60** 177.
Friedman J and Telegdi V L, 1957, *Phys Rev* **105** 1681.
Fujii H *et al.*, 1994, *Phys Rev D* **49** 559.
G R DeVoe, McIntyre P M, Magnon A, Stowell D Y, Swanson R A, and Telegdi V L, 1970, *Phys Rev Lett* **25** 1179.
Göppert M-Mayer, 1931, *Ann Phys* **9** 273.
Gabrielse G, 1981, *Phys Rev A* **23** 775.
Gardener C J, Badertscher A, Beer W, Bolton P R, Egan P O, Gladisch M, Greene M, Hughes V W, Lu D C, Mariam F G, Souder P A, Orth H, Vetter J, zu Putlitz G, 1982, *Phys Rev Lett* **48** 1168.
Garwin R L, Ledermann L M, and Weinrich W, 1957, *Phys Rev* **105** 1415.
Giacobino E and Cagnac B, 1980, in: Progress in Optics XVII, editor Wolf L (North Holland, Amsterdam) 85.
Glashow S, 1961, *Phys Rev Lett* **6** 196.
Gordeev V A *et al.*, 1994, *JETP Lett* **59** 589.
Grotch H and Hegstrom R A, 1971, *Phys Rev A* **4** 59.
Hänsch T W, 1977, *Soc Ital d Fis* **64** 17.
Halprin A and Masiero A, 1993, *Phys Rev D* **48** 2987.
Halprin A, 1982, *Phys Rev Lett* **48** 1313.
Hand L M, Miller D G, and Wilson R, 1963, *Rev Mod Phys* **35** 335.
Hauser P, Arb H P v, Biancchetti A, Hofer H, Kottmann F, Lüchinger C, Schaeren R, Studer F, and Unternährer J, 1992, *Phys Rev A* **46** 2363.
Hellwig H, Vessot R F C, Levine M W, Zitzewitz P, Allen D W, and Glaze D J, 1970, *IEEE Trans Instrum* **IM-19** 200.
Herczeg P and Mohapatra R N, 1992, *Phys Rev Lett* **69** 2475.
Herczeg P, 1997, Conference "Beyond the Desert 97", Castle Ringber.
Hinds A E, 1988, The Spectrum of Atomic Hydrogen Advances, editor Seriės G W, 245.
Horrikawa K and Sasaki K, 1996, *Phys Rev D* **53** 560.
Hou W S and Wong G G, 1996, *Phys Rev D* **53** 1537.
Hou W S, 1996, *Nucl Phys B* **51A** 40.
Huber A, Udem Th, Gross B, Reichert J, Kourogi M, Pachucki K, Weitz M, Hänsch T W, 1998, *Phys Rev Lett* **80** 468.
Huff R W, *Phys Rev* **186**, 1367 (1969)
Hughes V W and Kinoshita T, Muon Physics, editors Hughes V W and Wu C S (, Academic Press, New York) 11.
Hughes V W and Kuti J, 1983, *Ann Rev Nucl Sci* **33** 611.
Hughes V W and zu Putlitz G, 1990, *Quantum Electrodynamics* (World Scientific, Singapore,) editor Kinoshita T, 822.
Hughes V W, McColm D W, Ziock K, and Prepost R, 1960, *Phys Rev Lett* **5** 63.
Hughes V W, in: Atomic Physics Methods in Modern Research, editors Jungmann K, Kowalski J, Reinhard I, Traeger F, (Springer Verlag, Heidelberg), 21.
Hughes V W, 1992, *Z Phys C* **56** 35.
Janissen A C, Beer G A, Mason G R, Olin A, Huber T M, Kunselman A R, Bowen T, Halverson P G, Fry C A, Kendall K R, Marshall G M, and Warren J B, 1990, *Phys Rev A* **42** 161.
Jeckelmann B, Beer W, de Chambrier G, Elsenhans O, Giovanetti K L, A P F Goudsmit, Leisi H J, Nakada T, Piller O, Rüetschi A, and Schwitz W, 1986, *Nucl Phys A* **457** 709; Jeckelmann B, Goudsmit P F A, Leisi H J,1994, *Phys Lett B* **335** 326.
Jeffrey A M *et al.*, 1997, *IEEE Trans Instr Meas* **46** 264.

Jungmann K *et al.*, 1995, *Appl Phys B* **60** 159.
Jungmann K, 1994, *Atomic Physics 14* editors Wineland D, Wieman C, Smith D, (New York: AIP Press) 102.
Jungmann K, 1992a, Muonic Atoms and Molecules, editors Schaller L A and Petitjean C, (Birkhäuser-Verlag) 77.
Jungmann K, 1992b, *Z Phys C* **56** 59.
Jungmann K, Baird P E G, Barr J R M, Bressler C, Curley P F, Dixson R, Eaton G H, Ferguson A I, Geerds H, Hughes V W, Kenntner J, Lea S N, Maas F, Persaud M A, zu Putlitz G, Sandars P G H, Schwarz W, Toner W T, Towrie M, Woodman G, Zhang L, and Zhang Z, 1991, *Z Phys D* **21** 241.
Kammel P, Foroughi F, Petitjean C and Renker D, 1998, Letter of Intent to PSI; P Kammel, 1998, Proc EXAT98 conference, Ascona.
Karhenboim S, 1996a, *Z Phys* **D 36** 11.
Karshenboim S, 1996b, hep-ph/9608462 and 1994 hep-ph/9411356.
Karshenboim S G, Shelyuto V A, and Eides M I, 1992, *Sov J Nucl Phys* **55** 257.
Karshenboim S, 1997, hep-ph/9712347.
Karshenboim S, 1998, *Physica Scripta* **57** 213.
Karshenboim S, 1993, *Zh Eksp Teor Fiz* **103** 1105.
Kaulard J *et al.*, 1998, submitted for publication.
Kettell S H, 1990, Ph D thesis, Yale University.
Kinoshita T and Marciano W J, 1990, Quantum Electrodynamics, editor Kinoshita T (World Scientific, Singapore) 419.
Kinoshita T and Nio M, 1997, *Phys Rev* **D 55** 7867.
Kinoshita T and Yennie D R, 1990, Quantum Electrodynamics, editor Kinoshita T (World Scientific, Singapore) 1.
Kinoshita T, 1998, Proc Workshop on Frontiers in QED, Sadansky, Bulgaria (1998) and hep-ph/9808351.
Kinoshita T, 1995, *IEEE Trans Instr Meas* **44** 498.
Kinoshita T, 1996, *IEEE Trans Instr Meas* **44** 498; 1996, *Rep Prog Phys* **59** 1459; 1997, *IEEE Trans Instrum Meas* **46** 108.
Klapdor H and Hirsch M, 1997, *Z Phys A* **359** 361; Klapdor H V-Kleingrothaus, Proc Beyond the Desert Conference, 1998, (Institute of Physics Publishing, Bristol) 485.
Klempt E, 1989, The Hydrogen Atom, editors Bassani G F, Inguscio M, and Hänsch T W (Springer, Berlin, Heidelberg, New York) 211.
Kobayashi M and Maskawa T, 1973, *Prog Theor Phys* **49** 652.
Konopinski E J and Mahmoud H M, 1953, *Phys Rev* **92** 1045.
Kosmas T S, Leontaris G K, Vergados J D, 1994, *Prog Part Nucl Phys* **33** 397.
Krüger E, Nistler W and Weirauch W, 1995, *Metrologia* **32** 117 and 1997, *IEEE Trans Instrum Meas* **46** 101.
Kreussler S and Sizmann R, 1982, *Phys Rev B* **26** 520.
Kuang Y, Arnold K P, Chmely F, Echhause M, Hughes V W, Kane J R, Kettell S, Kim D H, Kumar K, Lu D C, Ni B, Matthias B, Orth H, zu G Putlitz, Schäfer H R, Souder P A, and Woodle K, 1987, *Phys Rev A* **35** 3172.
Kuno Y, 1998, private communication.
Kunze P, 1993, *Z Phys* **83** 1.
LAMPF proposal 1054: Ultrahigh Precision Measurements on Muonium Ground State: Hyperfine Structure and Muon Magnetic Moment, 1986, Hughes V W, zu Putlitz G, Souder P A, spokesmen.
Lamb W E and Retherford R C, 1950, *Phys Rev* **79** 549, and 1947, *Phys Rev* **71** 241.
Lanacker P, Luo M, Mann A K, 1992, *Rev Mod Phys* **64** 87.

Lepage G P and Yennie D R, 1984, iPrecision Measurement and Fundamental Constants II, editors Taylor B N and Phillips W D, (Nat Bur Stand, Washington) 185.

Lethokhov V S and Chebotaev V P, 1977, Nonlinear Laser Spectroscopy (Springer, Berlin, Heidelberg, New York).

Liu W, Boshier M G, Dhawan S, van Dyck O, Egan P, Fei X, Grosse M-Perdekamp, Hughes V W, Janousch M, Jungmann K, Kawall D, Mariam F G, Pillai C, Prigl R, zu Putlitz G, Reinhard I, W, Schwarz, Thompson P A, and Woodle K A, 1998, submitted to Phys Rev Lett; Liu W, 1997, PhD thesis YAle University.

Lopez J *et al.*, 1994, *Phys Rev D* **49** 366; Couture G and Konig H, 1996, *Phys Rev D* **53** 555; Chattopathyay U and Nath P, 1996, *Phys Rev D* **53** 1648; Moroi T, 1996, *Phys Rev D* **53** 6565.

Lundeen S R and Pipkin F M, 1986, *Metrologia* **22** 9.

Maas F, Baird P E G, Barr J R M, Berekeland D, Boshier M G, Braun B, Eaton G H, Ferguson A I, Geerds H, Hughes V W, Jungmann K, Matthias B M, Matousek P, Persaud M A, zu Putlitz G, Reinhard I, Riis E, Sandars P G H, Schwarz W, Toner W T, Towrie M, Willmann L, Woodle K A, Woodman G and Zhang L, 1994, *Phys Lett* **A187** 247; see also: Schwarz W, Baird P E G, Barr J R M, Berekeland D, Boshier M G, Braun B, Eaton G H, Ferguson A I, Geerds H, Hughes V W, Jungmann K, Maas F, Matthias B M, Matousek P, Persaud M A, zu Putlitz G, Reinhard I, Riis E, Sandars P G H, Toner W T, Towrie M, Willmann L, Woodle K A, Woodman G and Zhang L, 1995, *IEEE Trans Instr Meas* **44** 505.

Mariam F G, Beer W, Bolton P R, Egan P O, Gardner C J, Hughes V W, Lu D C, Souder P A, Orth H, Vetter J, Moser U, and zu Putlitz G, 1982, *Phys Rev Lett* **49** 993.

Martyn H U, 1990, in: Quantum Electrodynamics, editor Kinoshita T (World Scientific, Singapore) 92.

Matthias B E *et al.*, 1991, *Phys Rev Lett* **66** 2716.

Matthias B M, 1991, PhD thesis, Yale University.

Mery P *et al.*, 1990, *Z Phys C* **46** 229.

Meyer V *et al.*, 1997, loc cit Belz (1997), 429.

Miller J P *et al.*, 1997, loc cit Belz (1997), 792.

Mills A P, Imazato J, Kawashima Y, Saitoh S, Uedono A, and Nagamine K, 1986, *Phys Rev Lett* **56** 1463.

Mills A P and Chu S,1990, Quantum Electrodynamics, editor Kinoshita T, (World Scientific, Singapore) 774.

Missimer J and Simons L M, 1990, *Z Phys D* **17** 275.

Mohapatra R N, 1993, *Prog Part Nucl Phys* **31** 39.

Mohapatra R N, 1992, *Z Phys C* **56** 117.

Mohr P, 1988, The Spectrum of Atomic Hydrogen Advances, editor Series G W, 111.

Molzon W *et al.*, 1997, Proposal to BNL E-940.

Molzon W, 1998, JHF98 workshop, Tsukuba.

Mundinger H J, Arnold K P, Gladisch M, Hofmann J, Jacobs W, Orth H, zu G Putlitz, Rosenkranz J, Schäfer W, Schwarz W, Woodle K A, and Hughes V W, 1989, *Euro Phys Lett* **8** 339.

Nafe J E and Nelson E B, 1948, *Phys Rev* **71** 718.

Nafe J E, Nelson E B, and Rabi I I, 1947, *Phys Rev* **71** 914.

Nagamine K and Ishida K, 1992, Perspectives of Meson Science, editors Yamazaki T, Nakai K, and Nagamine K (North Holland, Amsterdam) 441.

Nagamine K *et al.*, 1998, RAL-Riken Experiment; K Nagamine, Scottish Summer School on Physics 51, St Andrews.

Nagamine K *et al.*, 1995, *Phys Rev Lett* **74** 4811.

Navarria F *et al.*, 1998, ETHZ-IPP PR-98-04; Kammel P, private communication.

Nistler W, 1998, private commication.

Oram C J, Bailey J M, Schmor P W, Fry C A, Kiefl R F, Warren J B, Marshall G M, and Olin A, 1984, *Phys Rev Lett* **52** 910.

Orth H, Arnold K P, Egan P O, Gladisch M, Jacobs W, Vetter J, Wahl W, Wiegand M, zu Putlitz G, Hughes V W, 1980, *Phys Rev Lett* **45** 1483.

Pachucki K and Karshenboim S, 1998, *Phys Rev Lett* **80** 2101.

Pachucki K, 1998, private communication; K Pachucki, 1998, workshop on QED and the Physics of the Vacuum, Sadansky.

Pachucki K, 1996, *Phys Rev A* **54** 1994.

Pachucki K, 1994, *Phys Rev Lett* **72** 3154.

Pachuki K, 1992, *Phys Rev A* **46** 648.

Palchikov V G, Sokolov Y L, and Yakovlev V P, 1985, *Metrologia* **21** 99; 1997, *Physica Scripta* **55** 33.

Palmer R B and Gallardo J C, 1998, physics/9802002; Palmer R B, 1998, physics/9802005.

Phillips J A, 1955, *Phys Rev* **97** 404.

Phillips W D *et al.*, 1977, *Metrologia* **13**, 81.

Pipkin F M, 1990, in: Quantum Electrodynamics, editor Kinoshita T (World Scientific, Singapore) 696.

Pontecorvo B, 1959, *Sov Phys -JETP* **37** 1751 and 1958, *Sov Phys JETP* **6** 381.

Pontecorvo B, 1958, *Sov Phys JETP* **6** 429.

Prepost R, Hughes V W, and Ziock K, 1961, *Phys Rev Lett* **6** 19.

Prigl R *et al.*, 1998, accepted for publication in IEEE Trans Instr Meas; see also: J P Miller *et al.*, 1997, Proceedings of "6th Conference on the Intersections of Particle and Nuclear Physics", editor Donnelly T W, AIP Conf Proc 412 429.

Prigl R *et al.*, 1996, Nucl Instr Meth A 304 349.

Raidal M and Santamaria A, 1997, hep-ph/9710389.

Raidal M, 1998, *Phys Rev D* **57** 2013.

Renard F M *et al.*, 1997, *Phys Lett B* **409** 398.

Roberts B L, 1992, *Z Phys C* **56** 101.

Rosenfelder R, 1992, Paul Scherrer Institute, Villigen, private communication.

Rosenfelder R, 1992, Muonic Atoms and Molecules, editors Schaller L and Petitjean C (Birkhäuser, Basel) 95.

J, Arnold K P, Gladisch M, Hofmann J, Mundinger H J, Orth H, zu Putlitz G, Stickel M, Schäfer W, Schwarz W, and Hughes V W, 1990, *Ann Phys* **47** 667.

Salpeter E E, 1952, *Phys Rev* **87** 328.

Sapirstein J R and Yennie D R, 1990, Quantum Electrodynamics, editor Kinoshita T, (World Scientific, Singapore) 560.

Schäfer W, 1988, doctoral dissertation, Heidelberg University.

Schaaf A V D *et al.*, 1998, Letter of Intent to PSI, R98-05 0.

Schaller L, 1992, *Z Phys C* **56** 48.

Scheck F, 1978, *Phys Rep* **44** 187.

Schrödinger E, 1926, *Ann d Phys* **79** 361.

Schultz P J and Lynn K G, 1988, *Rev Mod Phys* **60** 701.

Schwarz W and Hughes V W, 1993, private comunication.

Semertzidis Y *et al.*, 1997, Letter of Intent to BNL-AGS.

Shabaev V M, 1998, *J Phys B* **31** 337.

Sick I, 1982, *Phys Lett* **116B** 212.

Simon G G, Schmitt Ch, Barkowski F, and Walther V W, 1980, *Nucl Phys A* **333** 381.

Smith P W and Hänsch T W, 1971, *Phys Rev Lett* **26** 740.

Souder P A, Crane T W, Hughes V W, Lu D C, Orth H, Reist H W, Yam M H, zu Putlitz G, 1980, *Phys Rev A* **22** 33.

Springer M, 1993, Diplomarbeit, Heidelberg University and Matthias B E *et al.*, to be published.

Stambaugh R D, Casperson D E, Crane T W, Hughes V W, Kaspar H F, Souder P, Thompson P A, Orth H, zu Putlitz G, and Denison A B, 1974, *Phys Rev Lett* **33** 568.

Starshenko V V and Faustov R N, 1983, *Moscow Univ Phys Bull (USA)* **38** 55.

Steinberger J, 1991, *Phys Rep* **203** 345.

Stone J, 1998, Proceedings of the 1st Tropical Workshop on Particle Physics and Cosmology, editor Nieves J, (AIP Press).

Taqqu D *et al.*, 1998, PSI letter of intent.

Udem Th, Huber A, Gross B, Reichert J, Prevedelli M, Weitz M, and Hänsch T W Phys Rev Lett 79, 2646 (1997); see also: Huber A, Udem Th, Gross B, Reichert J, Kourogi M, Pachucki K, Weitz M, and Hänsch T W, 1998, Phys Rev Lett 80 468.

Thompson P A, Amato J J, Crane P, Hughes V W, Mobley R M, zu Putlitz G, and Rotheberg J E, 1968, *Phys Rev Lett* **22** 163.

Thompson P A, Crane P, Crane T, Amato J J, Hughes V W, zu Putlitz G, Rothenberg J E, 1973, *Phys Rev A* **8** 86.

Van Dyck R S Jr., 1990, Quantum Electrodynamics, editor Kinoshita T (World Scientific, Singapore) 322.

van Wijnjaarden A, Holuj F and Drake G, 1998, *Can Jour Phys* **76** 95.

Vasilenko L S, Chebotaev V P, Shishaev A V, 1970, *JETP Lett* **12** 113.

Vergados J D, 1986, *Phys Rep* **133** 122.

von Arb H P, Dittus F, Heeb H, Hofer H, Kottmann F, Niggli S, Schaeren R, Taqqu D, Unternährer J, and Egelhof P, 1984, *Phys Lett* **136B** 232.

Williams E R *et al.*, 1989, *IEEE Trans Instrum Meas* **38** 233.

Willmann L and Jungmann K, 1997, Lecture Notes in Physics, Vol 499.

Wong G G and Hou W S, 1995, *Phys Lett B* **357** 145.

Wong G G and Hou W S, 1994, *Phys Rev D* **50** R2962.

Woodle K A, Arnold K P, Gladisch M, Hofmann J, Janousch M, Jungmann K, Mundinger H J, zu Putlitz G, Rosenkranz J, Schäfer W, Schiff G, Schwarz W, Hughes V W and Kettell S H, 1988, *Z Phys D* **9** 59.

Yakhontov V and Jungmann K, 1996, *Z Phys* **D 38** 141.

Yakhontov V, Santra R and Jungmann K, 1998, submitted to *J Phys B.*

Yennie D R, 1992, *Z Phys C* **56** 13.

Zeldovitch Y B, 1952, *Dan SSR* **86** 505.

Zeller M, 1998, private communication ;see also Eilerts S, 1997, loc cit Belz (1997), 779)F Cuypers and Davidson S, 1998, *Eur Phys J* **C2** 503.